# AIEEE - 2008

## Answers by

**Aakash IIT-JEE**

(Division of Aakash Educational Services Ltd.)

| CODES | | | | |
|---|---|---|---|---|
| Q.No. | A-5 | B-5 | C-5 | D-5 |
| 001 | 3 | 2 | 2 | 2 |
| 002 | 3 | 2 | 2 | 2 |
| 003 | 2 | 3 | 3 | 2 |
| 004 | 3 | 3 | 1 | 1 |
| 005 | 2 | 1 | 2 | 4 |
| 006 | 3 | 3 | 3 | 1 |
| 007 | 3 | 2 | 4 | 2 |
| 008 | 1 | 4 | 3 | 1 |
| 009 | 2 | 3 | 4 | 2 |
| 010 | 1 | 1 | 4 | 2 |
| 011 | 4 | 4 | 4 | 4 |
| 012 | 2 | 3 | 4 | 2 |
| 013 | 3 | 3 | 4 | 1 |
| 014 | 1 | 4 | 4 | 4 |
| 015 | 2 | 3 | 3 | 3 |
| 016 | 3 | 3 | 2 | 2 |
| 017 | 3 | 4 | 2 | 3 |
| 018 | 2 | 2 | 4 | 3 |
| 019 | 1 | 1 | 2 | 4 |
| 020 | 4 | 1 | 1 | 4 |
| 021 | 1 | 3 | 1 | 1 |
| 022 | 2 | 4 | 3 | 2 |
| 023 | 3 | 2 | 2 | 2 |
| 024 | 3 | 1 | 1 | 4 |
| 025 | 3 | 1 | 2 | 1 |
| 026 | 2 | 4 | 2 | 4 |
| 027 | 1 | 1 | 1 | 3 |
| 028 | 1 | 1 | 3 | 4 |
| 029 | 4 | 1 | 2 | 1 |
| 030 | 1 | 1 | 4 | 2 |
| 031 | 3 | 4 | 3 | 4 |
| 032 | 1 | 3 | 4 | 3 |
| 033 | 1 | 4 | 4 | 4 |
| 034 | 4 | 1 | 3 | 1 |
| 035 | 4 | 1 | 4 | 2 |

| CODES | | | | |
|---|---|---|---|---|
| Q.No. | A-5 | B-5 | C-5 | D-5 |
| 036 | 3 | 1 | 4 | 1 |
| 037 | 4 | 3 | 1 | 2 |
| 038 | 1 | 2 | 4 | 3 |
| 039 | 2 | 3 | 3 | 1 |
| 040 | 2 | 4 | 2 | 4 |
| 041 | 4 | 3 | 4 | 3 |
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| 043 | 1 | 3 | 1 | 4 |
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| 045 | 2 | 1 | 2 | 2 |
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| 049 | 2 | 4 | 4 | 2 |
| 050 | 4 | 3 | 3 | 2 |
| 051 | 2 | 3 | 2 | 3 |
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| 053 | 3 | 2 | 3 | 3 |
| 054 | 4 | 4 | 3 | 2 |
| 055 | 1 | 1 | 3 | 4 |
| 056 | 3 | 2 | 2 | 3 |
| 057 | 1 | 2 | 1 | 2 |
| 058 | 3 | 1 | 4 | 3 |
| 059 | 1 | 4 | 3 | 2 |
| 060 | 4 | 1 | 1 | 1 |
| 061 | 3 | 3 | 3 | 4 |
| 062 | 1 | 4 | 4 | 3 |
| 063 | 2 | 3 | 2 | 1 |
| 064 | 1 | 2 | 4 | 2 |
| 065 | 2 | 4 | 1 | 2 |
| 066 | 2 | 3 | 3 | 4 |
| 067 | 3 | 2 | 2 | 1 |
| 068 | 4 | 1 | 4 | 1 |
| 069 | 1 | 4 | 1 | 4 |
| 070 | 3 | 4 | 3 | 1 |

| CODES | | | | |
|---|---|---|---|---|
| Q.No. | A-5 | B-5 | C-5 | D-5 |
| 071 | 4 | 1 | 2 | 2 |
| 072 | 3 | 2 | 2 | 4 |
| 073 | # | 4 | 4 | 2 |
| 074 | 4 | 2 | 3 | 3 |
| 075 | 1 | 4 | 1 | 1 |
| 076 | 1 | 1 | 2 | 3 |
| 077 | 2 | 3 | 1 | 1 |
| 078 | 4 | 1 | 2 | 2 |
| 079 | 4 | 3 | 3 | 3 |
| 080 | 3 | 4 | 4 | 1 |
| 081 | 3 | 2 | 1 | # |
| 082 | 2 | 4 | 4 | 1 |
| 083 | 2 | # | 3 | 3 |
| 084 | 1 | 1 | # | 3 |
| 085 | 3 | 2 | 4 | 1 |
| 086 | 1 | 4 | 2 | 3 |
| 087 | 1 | 4 | 4 | 2 |
| 088 | 4 | 1 | 1 | 1 |
| 089 | 4 | 3 | 1 | 2 |
| 090 | 2 | 2 | 3 | 4 |
| 091 | 4 | 2 | 3 | 4 |
| 092 | 3 | 3 | 3 | 4 |
| 093 | 2 | 3 | 3 | 2 |
| 094 | 4 | 2 | 2 | 2 |
| 095 | 2 | 4 | 1 | 3 |
| 096 | 2 | 3 | 4 | 4 |
| 097 | 3 | 1 | 4 | 4 |
| 098 | 2 | 2 | 2 | 1 |
| 099 | 1 | 1 | 1 | 3 |
| 100 | 3 | 4 | 1 | 1 |
| 101 | 2 | 3 | 3 | 1 |
| 102 | 4 | 3 | 3 | 2 |
| 103 | 1 | 1 | 2 | 3 |
| 104 | 1 | 4 | 4 | 4 |
| 105 | 3 | 2 | 1 | 4 |

**Though every care has been taken to provide the answers correctly but the Institute shall not be responsible for error, if any.**

# No Choice is correct

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# Solutions to AIEEE - 2008

**Time : 3 hrs.** **CODE - C5** **Max. Marks: 315**

**Instructions :**

1. The Test is of 3 hrs duration.
2. The Test booklet consists of 105 questions of 3 marks each. The maximum marks are 315.
3. There are three parts in the question paper.
4. The distribution of marks subjectwise in each part is as under for each correct response.
   Part-A - Mathematics (105 marks) - 35 questions
   Part-B - Chemistry (105 marks) - 35 questions
   Part-C - Physics (105 marks) - 35 questions
5. Candidates will be awarded **three** marks for indicated **correct** response of each question. **One** mark will be deducated fro indicated **incorrect** response of each question. **No deduction** from the total score will be made **if no response** is indicated for an item in the Answer Sheet.



## PART - A
## MATHEMATICS

1. *AB* is a vertical pole with *B* at the ground level and *A* at the top. A man finds that the angle of elevation of the point *A* from a certain point *C* on the ground is 60º. He moves away from the pole along the line *BC* to a point *D* such that *CD* = 7 m. From *D* the angle of elevation of the point *A* is 45º. Then the height of the pole is

(1) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}$ m

(2) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)$ m

(3) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)$ m

(4) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}+1}$ m

**Answer (2)**

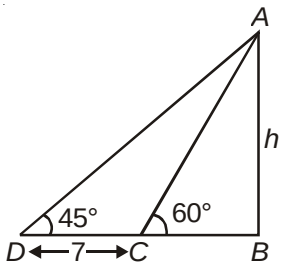
**Hints :** In $\Delta ABC$

$BC = h\cot 60°$

In $\Delta ABD$

$BD = h\cot 45°$

$BD - BC = DC$



$$\Rightarrow \quad h = \frac{7}{\cot 45° - \cot 60°} = \frac{7}{\left(1 - \frac{1}{\sqrt{3}}\right)} = \frac{7\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{7\sqrt{3}}{2} \times \left(\sqrt{3}+1\right)$$

2. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(A \mid B) = \frac{1}{2}$ and $P(B \mid A) = \frac{2}{3}$. Then $P(B)$ is

(1) $\frac{1}{6}$ (2) $\frac{1}{3}$

(3) $\frac{2}{3}$ (4) $\frac{1}{2}$

**Answer (2)**

**Hints :** $P(A/B) = \frac{P(A \cap B)}{P(B)}$ ...(1)

$P(B/A) = \frac{P(B \cap A)}{P(A)}$ ...(2)

$$\therefore \quad P(B) = \frac{P(B/A).P(A)}{P(A/B)} = \frac{\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{1}{3}$$

3. A die is thrown. Let A be the event that the number obtained is greater than 3. Let $B$ be the event that the number obtained is less than 5. Then $P(A \cup B)$ is

(1) $\frac{3}{5}$ (2) 0

(3) 1 (4) $\frac{2}{5}$

**Answer (3)**

**Hints :** $A = \{4, 5, 6\}$, $B = \{1, 2, 3, 4\}$, $A \cap B = \{1\}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{6} + \frac{4}{6} - \frac{1}{6} = 1$$

4. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is

(1) $\frac{8}{3}$ (2) $\frac{2}{3}$

(3) $\frac{4}{3}$ (4) $\frac{5}{3}$

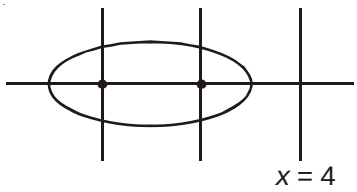
**Answer (1)**

**Hints :** $\frac{a}{e} - ae = 4$

$e = \frac{1}{2}$

$\therefore \quad 2a - \frac{a}{2} = 4 \Rightarrow \frac{3a}{2} = 4$

$a = \frac{8}{3}$



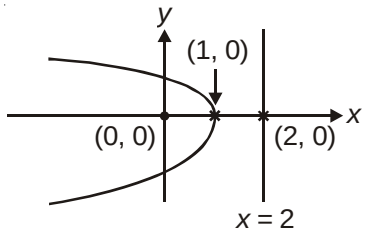
5. A parabola has the origin as its focus and the line $x = 2$ as the directrix. Then the vertex of the parabola is at

(1) (0, 2) (2) (1, 0)

(3) (0, 1) (4) (2, 0)

**Answer (2)**

**Hints :**




Vertex of the parabola at (1, 0)

6. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is

(1) (3, –4) (2) (–3, 4)

(3) (–3, –4) (4) (3, 4)

**Answer (3)**

**Hints :** $(x + 1)^2 + (y + 2)^2 = \left(2\sqrt{2}\right)^2$

Let required point be $Q(\alpha, \beta)$

Then midpoint of $P(1, 0)$ and $Q(\alpha, \beta)$ is centre of the circle

$\therefore \quad \frac{\alpha + 1}{2} = -1$ and $\frac{\beta + 0}{2} = -2$

$\Rightarrow \quad \alpha = -3, \beta = -4.$

7. Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$ where $Y = \{y \in N : y = 4x + 3$ for some $x \in N\}$. Show that $f$ is invertible and its inverse is

(1) $g(y) = \frac{3y + 4}{3}$ (2) $g(y) = 4 + \frac{y + 3}{4}$

(3) $g(y) = \frac{y + 3}{4}$ (4) $g(y) = \frac{y - 3}{4}$

**Answer (4)**

**Hints :** $y = 4x + 3 \Rightarrow x = \frac{y - 3}{4}$

8. The conjugate of a complex number is $\frac{1}{i - 1}$. Then that complex number is

(1) $\frac{-1}{i - 1}$ (2) $\frac{1}{i + 1}$

(3) $\frac{-1}{i + 1}$ (4) $\frac{1}{i - 1}$

**Answer (3)**

**Hints :** $\overline{\left(\frac{1}{i - 1}\right)} = \frac{1}{-i - 1} = \frac{-1}{i + 1}$

9. Let $R$ be the real line. Consider the following subsets of the plane $R \times R$ :

$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$

$T = \{(x, y) : x - y \text{ is an integer}\}$

Which one of the following is true ?

(1) Neither $S$ nor $T$ is an equivalence relation on $R$

(2) Both $S$ and $T$ are equivalence relations on $R$

(3) $S$ is an equivalence relation on $R$ but $T$ is not

(4) $T$ is an equivalence relation on $R$ but $S$ is not

**Answer (4)**

**Hints :** $(1, 2) \in S$

but $(2, 1) \notin S$

$\therefore$ $S$ is not symmetric; $\therefore$ not equivalent.

10. The perpendicular bisector of the line segment joining $P(1, 4)$ and $Q(k, 3)$ has $y$-intercept $-4$. Then a possible value of $k$ is

(1) 1 (2) 2

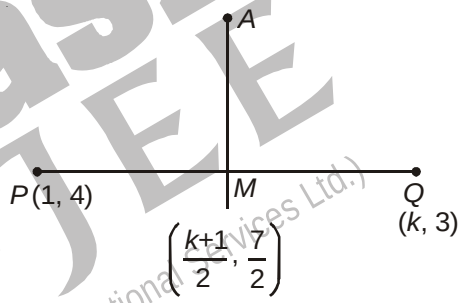
(3) –2 (4) –4

**Answer (4)**

**Hints :** Slope of $PQ = \dfrac{4-3}{1-k}$

$\therefore$ Equation of AM is

$$y - \frac{7}{2} = (k-1)\left[x - \left(\frac{k+1}{2}\right)\right]$$




For $y$-intercept, $x = 0, y = -4$

$$\therefore \quad -4 - \frac{7}{2} = -(k-1)\left(\frac{k+1}{2}\right)$$

$$\Rightarrow \quad \frac{15}{2} = \frac{k^2-1}{2} \Rightarrow k^2 - 1 = 15$$

$k^2 = 16$

$k = -4$.

11. The solution of the differential equation $\dfrac{dy}{dx} = \dfrac{x+y}{x}$ satisfying the condition $y(1) = 1$ is

(1) $y = \ln x + x$ (2) $y = x \ln x + x^2$

(3) $y = x\, e^{(x-1)}$ (4) $y = x \ln x + x$

**Answer (4)**

**Hints :** $\dfrac{dy}{dx} - \dfrac{1}{x} \cdot y = 1$

$$\text{I.F.} = e^{-\int \frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$$

$$\therefore \quad y\left(\frac{1}{x}\right) = \int \frac{1}{x} dx = \log x + c$$

$y(1) = 1 \Rightarrow 1 = \log 1 + c \Rightarrow c = 1$

$\therefore$ $y = x(\log x + 1) = x \log x + x$

12. The mean of the numbers *a*, *b*, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of *a* and *b*?

(1) $a = 0, b = 7$

(2) $a = 5, b = 2$

(3) $a = 1, b = 6$

(4) $a = 3, b = 4$

**Answer (4)**

**Hints :** $6.80 = \dfrac{(6-a)^2 + (6-b)^2 + (6-8)^2 + (6-5)^2 + (6-10)^2}{5}$

$\Rightarrow \quad 34 = (6-a)^2 + (6-b)^2 + 4 + 1 + 16$

$\Rightarrow \quad (6-a)^2 + (6-b)^2 = 13 = 9 + 4 = 3^2 + 2^2$

$a = 3, b = 4$

13. The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of $\alpha$ and $\beta$?

(1) $\alpha = 2, \beta = 2$

(2) $\alpha = 1, \beta = 2$

(3) $\alpha = 2, \beta = 1$

(4) $\alpha = 1, \beta = 1$

**Answer (4)**

**Hints :** $\vec{a}, \vec{b}, \vec{c}$ are coplanar

$\therefore \begin{vmatrix} \alpha & 2 & \beta \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 0$

$\Rightarrow \quad \alpha(1-0) - 2(1-0) + \beta(1-0) = 0$

$\Rightarrow \quad \alpha + \beta = 2$

14. The non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is

(1) 0 (2) $\dfrac{\pi}{4}$

(3) $\dfrac{\pi}{2}$ (4) $\pi$

**Answer (4)**

**Hints :** $\vec{a}$ is parallel to $\vec{b}$, $\vec{c}$ is anti-parallel to $\vec{b}$

$\therefore \quad \vec{a}$ and $\vec{c}$ are anti-parallel.

$\therefore \quad$ Angle between $\vec{a}$ and $\vec{c}$ is $\pi$.

15. The line passing through the points (5, 1, *a*) and (3, *b*, 1) crosses the *yz*-plane at the point $\left(0, \dfrac{17}{2}, \dfrac{-13}{2}\right)$. Then

(1) $a = 2, b = 8$ (2) $a = 4, b = 6$

(3) $a = 6, b = 4$ (4) $a = 8, b = 2$

**Answer (3)**

**Hints :** Equation of line through (5, 1, $a$) and (3, $b$, 1).

$$\frac{x-3}{5-3} = \frac{y-b}{1-b} = \frac{z-1}{a-1} \quad ...(1)$$

$\left(0, \frac{17}{2}, -\frac{13}{2}\right)$ satisfies (1)

$$\therefore \quad \frac{-3}{2} = \frac{\frac{17}{2} - b}{1-b} = \frac{\frac{-13}{2} - 1}{a-1}$$

$$\therefore \quad a - 1 = \frac{\left(\frac{-15}{2}\right)}{\left(\frac{-3}{2}\right)} = 5 \Rightarrow a = 6$$

Also, $-3(1-b) = 2\left(\frac{17}{2} - b\right)$

$\Rightarrow \quad 3b - 3 = 17 - 2b \Rightarrow 5b = 20, b = 4$

16. If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ intersect at a point, then the integer $k$ is equal to

(1) –5 (2) 5

(3) 2 (4) –2

**Answer (2)**

**Hints :** Let $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3} = r$ ...(1)

and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2} = s$ ...(2)

Any point on (1) is $(1 + rk, 2 + 2r, 3 + 3r)$

and on (2) is $(2 + 3s, 3 + ks, 1 + 2s)$

If (1) & (2) intersect at a point, then

$1 + rk = 2 + 3s$, $2 + 2r = 3 + ks$ and $3 + 3r = 1 + 2s$

Adding all three equation

$6 + r(k + 2 + 3) = 6 + s(3 + k + 2)$

$\Rightarrow \quad r = s$

$\therefore \quad 3 + 3r = 1 + 2r \Rightarrow \quad r = -2$

$\therefore \quad 1 - k = 2 - 6 = -4, k = 5$

***Directions :*** *Questions number 17 to 21 are Assertion-Reason type questions. Each of these questions contains two statements. Statement-1 (Assertion) and Statement-2 (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.*

17. Statement-1 : For every natural number $n \geq 2$, $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + .... + \frac{1}{\sqrt{n}} > \sqrt{n}$.

Statement-2 : For every natural number $n \geq 2$, $\sqrt{n(n+1)} < n + 1$.

(1) Statement-1 is false, Statement-2 is true

(2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is false

**Answer (2)**

**Hints :** We have,

$n(n+1) = n^2 + n < n^2 + n + n + 1 = (n+1)^2$

$\Rightarrow \quad \sqrt{n(n+1)} < n+1 \;\forall\; n \geq 2$

$\Rightarrow \quad \sqrt{n} < \sqrt{n+1}$

$\Rightarrow \quad \frac{1}{\sqrt{n}} < \frac{1}{\sqrt{n+1}} \qquad n \geq 2$

Statement 2 is true.

Also $\frac{1}{\sqrt{1}} > \frac{1}{\sqrt{n}}$

$\frac{1}{\sqrt{2}} > \frac{1}{\sqrt{n}}$

$\frac{1}{\sqrt{3}} > \frac{1}{\sqrt{n}}$

.....

$\frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n}}$

On adding them

$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ..... + \frac{1}{\sqrt{n}} > \frac{n}{\sqrt{n}} = \sqrt{n}$

Clearly statements 1 and 2 are true and statement 2 is a correct explanation of statement 1.

18. Let $A$ be a $2 \times 2$ matrix with real entries. Let $I$ be the $2 \times 2$ identity matrix. Denote by tr($A$), the sum of diagonal entries of $A$. Assume that $A^2 = I$.

Statement-1 : If $A \neq I$ and $A \neq -I$, then det $A = -1$.

Statement-2 : If $A \neq I$ and $A \neq -I$, then tr($A$) $\neq 0$.

(1) Statement-1 is false, Statement-2 is true

(2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is false

**Answer (4)**

**Hints :** $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \quad \left.\begin{matrix} b(a+d)=0 \\ c(a+d)=0 \end{matrix}\right\} \qquad a = -d\,, \qquad a^2 + bc = 1$

If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$A \neq I, A \neq -I$

det $A = -1$ (Statement-1 is true)

Statement-2, $tr(A) = 1 - 1 = 0$, Statement-2 is (false)

19. Statement-1 : $\sum_{r=0}^{n}(r+1)\ {}^{n}C_r = (n+2)\ 2^{n-1}$.

Statement-2 : $\sum_{r=0}^{n}(r+1)\ {}^{n}C_r x^r = (1+x)^n + nx(1+x)^{n-1}$.

(1) Statement-1 is false, Statement-2 is true
(2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(3) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1
(4) Statement-1 is true, Statement-2 is false

**Answer (2)**

**Hints :** $\sum_{r=0}^{n} {}^{n}C_r \cdot x^r = (1+x)^n$

Multiplying by '$x$'

$\sum_{r=0}^{n} {}^{n}C_r \cdot x^{r+1} = x(1+x)^n$

Differentiating w.r.t. '$x$'

$\sum_{r=0}^{n}(r+1)\ {}^{n}C_r \cdot x^n = (1+x)^n + nx(1+x)^{n-1}$

$\therefore$ Statement-2 is True.

If $x = 1$, then

$\sum_{r=0}^{n}(r+1)\ {}^{n}C_r = 2^n + n(2)^{n-1} = (n+2)2^{n-1}$

$\therefore$ Statement-1 is true, and Statement-2 is a correct explanation of Statement-1.

20. Let $p$ be the statement "$x$ is an irrational number", $q$ be the statement "$y$ is a transcendental number", and $r$ be the statement "x is a rational number iff $y$ is a transcendental number".

Statement-1 : $r$ is equivalent to either $q$ or $p$.

Statement-2 : $r$ is equivalent to $\sim(p \leftrightarrow \sim q)$.

(1) Statement-1 is false, Statement-2 is true
(2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(3) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1
(4) Statement-1 is true, Statement-2 is false

**Answer (1)**

**Hints :** Clearly statement-1 is false.

21. In a shop there are five types of ice-creams available. A child buys six ice-creams.

Statement -1 The number of different ways the child can buy the six ice-creams is $^{10}C_5$.

Statement-2 The number of different ways the child can buy the six icd-creams is equal to the number of different ways of arranging 6 As and 4 B's in a row.

(1) Statement-1 is false, Statement-2 is true
(2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(3) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1
(4) Statement-1 is true, Statement-2 is false

**Answer (1)**

**Hints :** The number of ways that child can buy the six ice creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row

$= \frac{10!}{6!\ 4!} = {}^{10}C_4$

= number of integral solution of the equation $(x_1 + x_2 + x_3 + x_4 + x_5 = 6) = {}^{6+5-1}C_{5-1} = {}^{10}C_4 \neq {}^{10}C_5$

Statement 1 is false and statement 2 is true.

22. Let $f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1} & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$

The which one of the following is true?

(1) $f$ is neither differentiable at $x = 0$ nor at $x = 1$

(2) $f$ is differentiable at $x = 0$ and at $x = 1$

(3) $f$ is differentiable at $x = 0$ but not at $x = 1$

(4) $f$ is differentiable at $x = 1$ but not at $x = 0$

**Answer (3)**

**Hints :** $f'(1^-) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h}$

$= \lim_{h\to 0} \frac{(1-h-1)\cdot\sin\left(\frac{1}{1-h-1}\right)-0}{-h}$

$= \lim_{h\to 0} \sin\left(-\frac{1}{h}\right) = -\lim_{h\to 0} \sin\frac{1}{h}$

$f'(1^+) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$= \lim_{h\to 0} \frac{(1+h-1)\sin\left(\frac{1}{1+h-1}\right)-0}{h} = \lim_{h\to 0} \sin\frac{1}{h}$

$f'(1^-) \neq f'(1^+)$ $\therefore$ $f$ is not differentiable at $x = 1$

$f'(0^-) = \lim_{h\to 0} \frac{(0-h-1)\sin\left(\frac{1}{0-h-1}\right)-(-1)\cdot\sin(-1)}{-h}$

$= \lim_{h\to 0} \frac{(h+1)\sin\left(\frac{1}{h+1}\right)-\sin 1}{-h}$ ; $\left(\frac{0}{0}\right)$

$= \lim_{h\to 0} \frac{-\left\{(h+1)\cos\left(\frac{1}{h+1}\right)\left(\frac{1}{(h+1)^2}\right)\right\}+\sin\left(\frac{1}{h+1}\right)}{-1}$ ; Using L - Hospitals rule)

= cos 1 – sin 1

$f'(0^+) = \lim_{h\to 0} \frac{(0+h-1)\sin\left(\frac{1}{0+h-1}\right)-\sin 1}{h}$ ; $\left(\frac{0}{0}\right)$

$= \lim_{h\to 0} (h-1)\cos\left(\frac{1}{h-1}\right)\left(\frac{-1}{(h-1)^2}\right)+\sin\left(\frac{1}{h-1}\right)$ = cos 1 – sin 1

$f'(0^-) = f'(0^+)$ $\therefore$ $f$ is differentiable at $x = 0$.

23. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is

(1) –4 (2) – 12

(3) 12 (4) 4

**Answer (2)**

**Hints :** $a + ar = 12 = a(1 + r)$

$ar^2 + ar^3 = ar^2(1 + r) = 48$

$\Rightarrow \quad r^2 = 4 \quad \Rightarrow \quad r = -2$

$\therefore \quad a = -12$

24. Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then which one of the following holds?

(1) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$

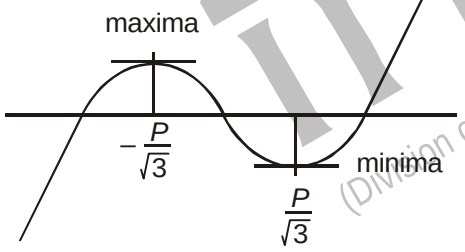
(2) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$

(3) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

(4) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

**Answer (1)**

**Hints :** $f'(x) = 3x^2 - p$




$f'(x) = 0 \quad \Rightarrow \quad x = \frac{p}{\sqrt{3}}, -\frac{p}{\sqrt{3}}$

25. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?

(1) 7 (2) 1

(3) 3 (4) 5

**Answer (2)**

**Hints :** $f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560$

$f'(x) = 7x^6 + 70x^4 + 48x^2 + 30 > 0 \quad \forall\ x \in R$

$\therefore \quad f(x)$ is increasing

$\therefore \quad f(x) = 0$ has only one solution

26. The statement $p \rightarrow (q \rightarrow p)$ is equivalent to

(1) $p \rightarrow (p \rightarrow q)$ (2) $p \rightarrow (p \vee q)$

(3) $p \rightarrow (p \wedge q)$ (4) $p \rightarrow (p \leftrightarrow q)$

**Answer (2)**

**Hints :**

| $q$ | $p$ | $p \to p$ | $p \to (q \to p)$ | $(p \vee q)$ | $p \to (p \vee q)$ |
|---|---|---|---|---|---|
| $T$ | $T$ | $T$ | $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ | $T$ | $T$ | $T$ |
| $F$ | $T$ | $T$ | $T$ | $T$ | $T$ |
| $F$ | $F$ | $T$ | $T$ | $F$ | $T$ |

$\therefore$ Statement $p \to (q \to q)$ is equivalent to $p \to (p \vee q)$.

27. The value of $\cot\left(\text{cosec}^{-1}\frac{5}{3} - \tan^{-1}\frac{2}{3}\right)$ is

(1) $\frac{6}{17}$ (2) $\frac{3}{17}$

(3) $\frac{4}{17}$ (4) $\frac{5}{17}$

**Answer (1)**

**Hints :** $\text{cosec}^{-1}\left(\frac{5}{3}\right) = \tan^{-1}\frac{3}{4}$

$\cot\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$


$= \cot\left|\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{1}{2}}\right| = \cot\ \tan^{-1}\left[\frac{\left(\frac{17}{12}\right)}{\left(\frac{1}{2}\right)}\right]$

$= \cot\ \tan^{-1}\left(\frac{17}{6}\right) = \frac{6}{17}$

28. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is

(1) $(x-2)y'^2 = 25 - (y-2)^2$ (2) $(y-2)y'^2 = 25 - (y-2)^2$

(3) $(y-2)^2\, y'^2 = 25 - (y-2)^2$ (4) $(x-2)^2\, y'^2 = 25 - (y-2)^2$

**Answer (3)**

**Hints :** $(x-\alpha)^2 + (y-2)^2 = 5^2$

$x^2 + \alpha^2 - 2\alpha x + y^2 + 4 - 4y = 25$

On differentiating

$2x - 2\alpha + 2y\frac{dy}{dx} - 4\frac{dy}{dx} = 0$

$\Rightarrow \quad \alpha = x + \frac{dy}{dx}(y-2) = 0$

$\left(x - x - \frac{dy}{dx}(y-2)\right)^2 + (y-2)^2 = 5^2$

$\Rightarrow \left(\frac{dy}{dx}\right)^2 (y-2)^2 = 25 - (y-2)^2$

$\Rightarrow \quad y'^2(y-2)^2 = 25 - (y-2)^2$

29. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}}\,dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}\,dx$. Then which one of the following is true?

(1) $I > \frac{2}{3}$ and $J > 2$ (2) $I < \frac{2}{3}$ and $J < 2$

(3) $I < \frac{2}{3}$ and $J > 2$ (4) $I > \frac{2}{3}$ and $J < 2$

**Answer (2)**

**Hints :** $I = \int_0^1 \frac{\sin x}{\sqrt{x}}dx < \int_0^1 \frac{x}{\sqrt{x}}dx$, because in $x \in (0, 1)$, $x > \sin x$

$$I < \int_0^1 \sqrt{x}\,dx = \frac{2}{3}\left[x^{3/2}\right]_0^1$$

$$\Rightarrow \quad I < \frac{2}{3}$$

$$J = \int_0^1 \frac{\cos x}{\sqrt{x}}dx < \int_0^1 x^{-\frac{1}{2}}dx = 2$$

$J < 2$

30. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to

(1) $\frac{5}{3}$ (2) $\frac{1}{3}$

(3) $\frac{2}{3}$ (4) $\frac{4}{3}$

**Answer (4)**

**Hints :** Solving, $x + 3y^2 = 1$ ...(1)

and $x + 2y^2 = 0$ ...(2)

$y^2 = 1$, $y = \pm 1$

Required area $= \left|\int_{-1}^{1}(x_1 - x_2)\,dy\right|$

$$= \left|\int_{-1}^{1}(1 - 3y^2 - 2y^2)\,dy\right|$$

$$= \left|\int_{-1}^{1}(1 - 5y^2)\,dy\right|$$

$$= \left|2\int_{0}^{1}(1 - 5y^2)\,dy\right|$$

$$= \left|2\left(y - \frac{5y^3}{3}\right)_0^1\right|$$

$$= \left|2\left(1 - \frac{5}{3}\right)\right| = \frac{4}{3}$$

31. The value of $\sqrt{2}\int \frac{\sin x\, dx}{\sin\left(x-\frac{\pi}{4}\right)}$ is

(1) $x+\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$ (2) $x-\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(3) $x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$ (4) $x-\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

**Answer (3)**

**Hints :** $x-\frac{\pi}{4}=t,\ dx=dt$

$$\therefore\ I=\sqrt{2}\int\frac{\sin\left(\frac{\pi}{4}+t\right)dt}{\sin t}$$

$$=\sqrt{2}\int\left[\frac{1}{\sqrt{2}}\cot t+\frac{1}{\sqrt{2}}\right]dt$$

$$=t+\log|\sin t|+c$$

$$=x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$$

32. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?

(1) 8. $^6C_4$. $^7C_4$ (2) 6.7. $^8C_4$

(3) 6. 8. $^7C_4$ (4) 7. $^6C_4$. $^8C_4$

**Answer (4)**

**Hints :** MISSISSIPPI

I = 4 times, S = 4 times, P = 2 times

_ M _ I _ I _ I _ I _ P _ P _

$$^8C_4\times\frac{7!}{4!\,2!}$$

$$^8C_4\times7\times\frac{6!}{4!\,2!}=7\cdot{}^8C_4\cdot{}^6C_4$$

33. Let $a$, $b$, $c$ be any real numbers. Suppose that there are real numbers $x$, $y$, $z$ not all zero such that $x = cy + bz$, $y = az + cx$, and $z = bx - ay$ Then $a^2 + b^2 + c^2 + 2abc$ is equal to

(1) 2 (2) – 1

(3) 0 (4) 1

**Answer (4)**

**Hints :** $x - cy - bz = 0$

$cx - y + az = 0$

$bx + ay - z = 0$

for non-zero solution

$$\begin{vmatrix}1 & -c & -b\\ c & -a & a\\ b & +a & -1\end{vmatrix}=0$$

$1(1-a^2)+c(-c-ab)-b(ac+b)=0$

$1-a^2-c^2-abc-abc-b^2=0$

$a^2+b^2+c^2+2abc=1$

34. Let A be a square matrix all of whose entries are intergers. The which one of the follwing is true ?
(1) If det A = ± 1, then $A^{-1}$ exists but all its intries are not necessarily integers
(2) If det A ≠ ± 1, then $A^{-1}$ exists and all its entries are non-integers
(3) If det A = ± 1, then $A^{-1}$ exists and all its entries are integers
(4) If det A = ± 1, then $A^{-1}$ neet not exist

**Answer (3)**

**Hints :** As det $A = \pm 1$, $A^{-1}$ exists and $A^{-1} = \frac{1}{\det A}(\text{adj } A) = \pm(\text{adj } A)$

All entries in adj $A$ are integers

$\therefore$ $A^{-1}$ has integer entries.

35. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is
(1) 1 (2) 4
(3) 3 (4) 2

**Answer (4)**

**Hints :** $x^2 - 6x + a = 0$ $(\alpha, 4\beta)$

$x^2 - cx + 6 = 0$ $(\alpha, 3\beta)$

$\alpha + 4\beta = 6$ $4\alpha\beta = a$ $\therefore \frac{a}{6} = \frac{4}{3}$

$\alpha + 3\beta = c$ $3\alpha\beta = 6$ $a = 8$

$x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$ $x = 2, 4$

$x^2 - cx + 6 = 0$

$2^2 - 2c + 6 = 0$ $c = 5$

$\therefore x^2 - 5x + 6 = 0 \Rightarrow x = 2, 3$

Common root is 2.



## PART - B
## CHEMISTRY

36. The organic chloro compound, which shows complete stereochemical inversion during a $S_N2$ reaction, is
(1) $(C_2H_5)_2CHCl$ (2) $(CH_3)_3CCl$
(3) $(CH_3)_2CHCl$ (4) $CH_3Cl$

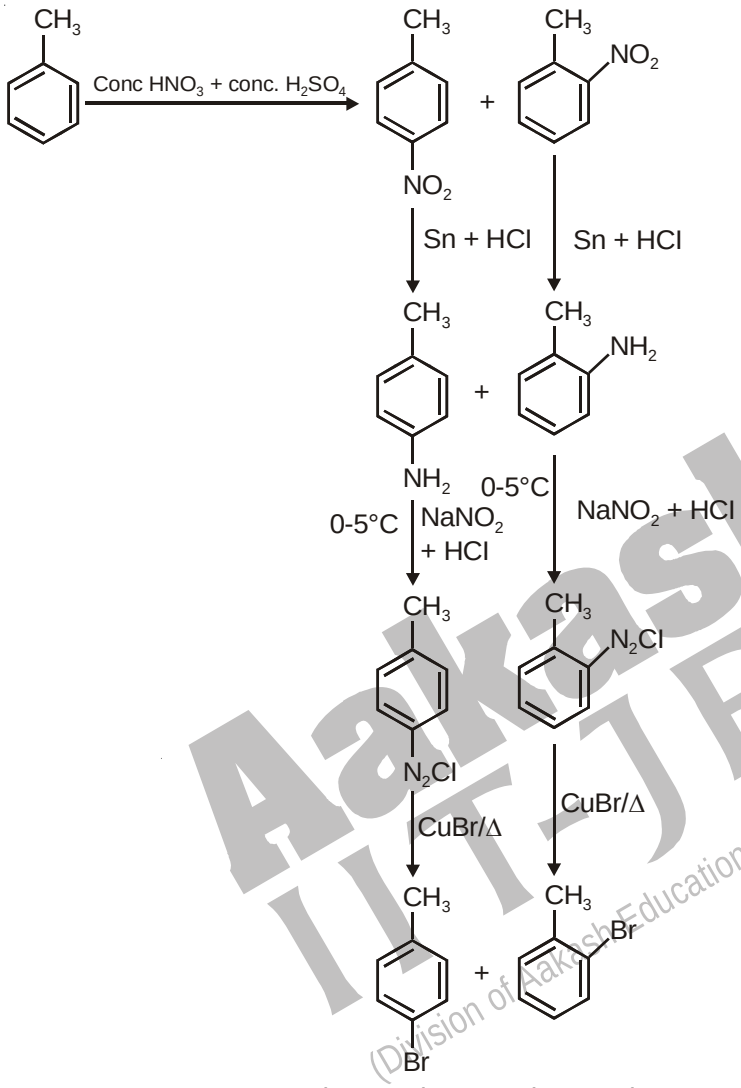
**Answer (4)**

**Hints :** $CH_3 - Cl$ shows complete stereochemical inversion because substitution takes place through $S_N2$ mechanism.

37. Toluene is nitrated and the resulting product is reduced with tin and hydrochloric acid. The product so obtained is diazotised and then heated with cuprous bromide. The reaction mixture so formed contains
(1) Mixture of *o*-and *p*-bromotoluenes
(2) Mixture of *o*-and *p*-dibromobenzenes
(3) Mixture of *o*-and *p*-bromoanilines
(4) Mixture of *o*-and *m*-bromotoluenes

**Answer (1)**

**Hints :**






38. The coordination number and the oxidation state of the element 'E' in the complex $[E(en)_2(C_2O_4)]NO_2$ (where (en) is ethylene diamine) are, respectively

(1) 6 and 2 (2) 4 and 2

(3) 4 and 3 (4) 6 and 3

**Answer (4)**

**Hints :** In complex $[E(en)_2(C_2O_4)]NO_2$ the co-ordination number of E is 6 and oxidation number is +3.

39. Identify the wrong statement in the following

(1) Chlorofluorocarbons are responsible for ozone layer depletion

(2) Greenhouse effect is responsible for global warming

(3) Ozone layer does not permit infrared radiation from the sun to reach the earth

(4) Acid rain is mostly because of oxides of nitrogen and sulphur

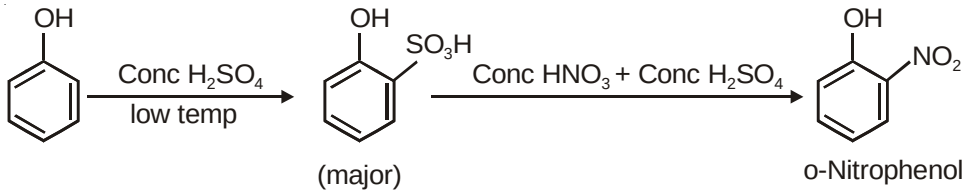
**Answer (3)**

**Hints :** Ozone layer does not permit UV radiation from the sun to reach the earth.

40. Phenol, when it first reacts with concentrated sulphuric acid and then with concentrated nitric acid, gives
(1) 2, 4, 6-trinitrobenzene (2) o-nitrophenol
(3) p-nitrophenol (4) Nitrobenzene

**Answer (2)**

**Hints :**




(major) o-Nitrophenol

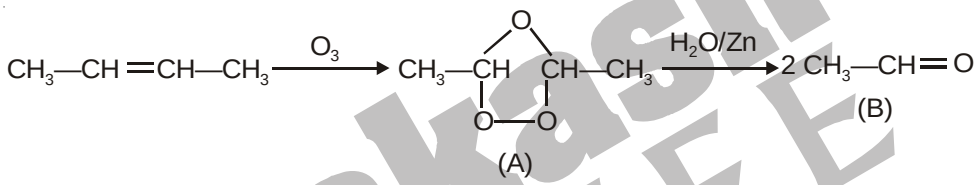
41. In the following sequence of reactions, the alkene affords the compound 'B'

$$CH_3CH=CHCH_3 \xrightarrow{O_3} A \xrightarrow[Zn]{H_2O} B.$$

The compound B is
(1) $CH_3CH_2CHO$ (2) $CH_3COCH_3$
(3) $CH_3CH_2COCH_3$ (4) $CH_3CHO$

**Answer (4)**

**Hints :**




(A) (B)

42. Larger number of oxidation states are exhibited by the actinoids than those by the lanthanoids, the main reason being
(1) 4*f* orbitals more diffused than the 5*f* orbitals
(2) Lesser energy difference between 5*f* and 6*d* than between 4*f* and 5*d* orbitals
(3) More energy difference between 5*f* and 6*d* than between 4*f* and 5*d* orbitals
(4) More reactive nature of the actinoids than the lanthanoids

**Answer (2)**

**Hints :** Because of lesser energy difference between 5*f* and 6*d* orbitals, electrons of these two orbitals may participate in bonding.

43. In which of the following octahedral complexes of Co(at. no. 27), will the magnitude of $\Delta_0$ be the highest?
(1) $[Co(CN)_6]^{3-}$ (2) $[Co(C_2O_4)_3]^{3-}$
(3) $[Co(H_2O)_6]^{3+}$ (4) $[Co(NH_3)_6]^{3+}$

**Answer (1)**

**Hints :**

$\Delta_0$ depends upon the field produced by ligand and charge on metal ion in octahedral complex. $CN^-$ is strong field ligand among all the given ligands therefore $\Delta_0$ will be highest for $[Co(CN)_6]^{3-}$

44. At 80°C, the vapour pressure of pure liquid 'A' is 520 mm Hg and that of pure liquid 'B' is 1000 mm Hg. If a mixture solution of 'A' and 'B' boils at 80°C and 1 atm pressure, the amount of 'A' in the mixture is (1 atm = 760 mm Hg)
(1) 52 mol percent (2) 34 mol percent
(3) 48 mol percent (4) 50 mol percent

**Answer (4)**

**Hints :**

$$P_{total} = X_A \cdot P°_A + X_B \cdot P°_B$$

$$760 = X_A \cdot 520 + (1 - X_A)\,1000$$

$$X_A = 0.5$$

mole percent of A = 50

45. For reaction $\frac{1}{2}A \rightarrow 2B$, rate of disappearance of A is related to the rate of appearance of 'B' by the expression

(1) $-\frac{d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt}$ (2) $-\frac{d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$

(3) $-\frac{d[A]}{dt} = \frac{d[B]}{dt}$ (4) $-\frac{d[A]}{dt} = 4\frac{d[B]}{dt}$

**Answer (2)**

**Hints :** $\frac{1}{2}A \longrightarrow 2B$

Rate law expression

$$-2\frac{d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt}$$

$$-\frac{d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$$

46. The equilibrium constants $K_{P_1}$ and $K_{P_2}$ for the reactions $X \rightleftharpoons 2Y$ and $Z \rightleftharpoons P + Q$, respectively are in the ratio of 1 : 9. If the degree of dissociation of X and Z be equal then the ratio of total pressure at these equilibria is

(1) 1 : 36 (2) 1 : 1

(3) 1 : 3 (4) 1 : 9

**Answer (1)**

**Hints :**

| | $X$ | $\rightleftharpoons$ | $2Y$ |
|---|---|---|---|
| initial mol | 1 | | 0 |
| at equilibrium | $1-\alpha$ | | $2\alpha$ |

total moles at equilibrium = $(1 + \alpha)$

let the total pressure is $P_1$ then

$$K_{P_1} = \frac{\left(\frac{2\alpha}{1+\alpha}.P_1\right)^2}{\left(\frac{1-\alpha}{1+\alpha}.P_1\right)}$$

for second reaction

| | $Z$ | $\rightleftharpoons$ | $P$ | + | $Q$ |
|---|---|---|---|---|---|
| Initial moles | 1 | | 0 | | 0 |
| at equilibrium | $1-\alpha$ | | $\alpha$ | | $\alpha$ |

total moles at equilibirum = $(1 + \alpha)$

Let total pressure is $P_2$

$$K_{P_2} = \frac{\left(\frac{\alpha}{1+\alpha}.P_2\right)\left(\frac{\alpha}{1+\alpha}.P_2\right)}{\left(\frac{1-\alpha}{1+\alpha}P_2\right)}$$

$$\frac{K_{P_1}}{K_{P_2}} = \frac{4P_1}{P_2} = \frac{1}{9}$$

$\therefore$ $P_1 : P_2 = 1 : 36$

47. Oxidising power of chlorine in aqueous solution can be determined by the parameters indicated below :

$$\frac{1}{2}Cl_2(g) \xrightarrow{\frac{1}{2}\Delta_{diss}H^{\ominus}} Cl(g) \xrightarrow{\Delta_{eg}H^{\ominus}} Cl^-(g) \xrightarrow{\Delta_{hyd}H^{\ominus}} Cl^-(aq)$$

The energy involved in the conversion of $\frac{1}{2}Cl_2(g)$ to $Cl^-(aq)$

(using the data $\Delta_{diss}H^{\ominus}_{Cl_2} = 240\ kJ\ mol^{-1}$, $\Delta_{eg}H^{\ominus}_{Cl} = -349\ kJ\ mol^{-1}$, $\Delta_{hyd}H^{\ominus}_{Cl^-} = -381\ kJ\ mol^{-1}$) will be

(1) +152 kJ mol$^{-1}$ (2) –610 kJ mol$^{-1}$

(3) –850 kJ mol$^{-1}$ (4) +120 kJ mol$^{-1}$

**Answer (2)**

**Hints :** $\frac{1}{2}Cl_2(g) \longrightarrow Cl^-(g)\ \ \Delta_{diss}\Delta H^{\ominus}Cl_2 = \frac{240}{2}\ kJ\ mol^{-1}$ .....(1)

$Cl(g) + e \longrightarrow Cl^-(g)\ \ \Delta_{eg}H^{\ominus}_{Cl} = -349\ kJ\ mol^{-1}$ .....(2)

$Cl^-(g) \longrightarrow Cl^-(aq)\ \ \Delta_{hyd}H^{\ominus}_{Cl^-} = -381\ kJ\ mol^{-1}$ .....(3)

Adding equation (1), (2) and (3), we get $\frac{1}{2}Cl_2(g) \longrightarrow Cl^-(aq)$

$\therefore\ \Delta H = \Delta_{diss}H^{\ominus}Cl_2 + \Delta_{eg}H^{\ominus}_{Cl} + \Delta_{hyd}H^{\ominus}_{Cl^-}$

$= \left(\frac{240}{2} - 349 - 381\right) kJ\ mol^{-1}$

$= -610\ kJ\ mol^{-1}$

48. Which of the following factors is of **no significance** for roasting sulphide ores to the oxides and not subjecting the sulphide ores to carbon reduction directly?

(1) Metal sulphides are thermodynamically more stable than $CS_2$

(2) $CO_2$ is thermodynamically more stable than $CS_2$

(3) Metal sulphide are less stable than the corresponding oxides

(4) $CO_2$ is more volatile than $CS_2$

**Answer (4)**

49. Bakelite is obtained from phenol by reacting with

(1) $(CH_2OH)_2$ (2) $CH_3CHO$

(3) $CH_3COCH_3$ (4) HCHO

**Answer (4)**

**Hints :** Bakelite is condensation polymer of phenol and formaldehyde

50. For the following three reactions a, b and c, equilibrium constants are given

a. $CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g)$; $K_1$

b. $CH_4(g) + H_2O(g) \rightleftharpoons CO(g) + 3H_2(g)$; $K_2$

c. $CH_4(g) + 2H_2O(g) \rightleftharpoons CO_2(g) + 4H_2(g)$; $K_3$

Which of the following relations is correct?

(1) $K_1\sqrt{K_2} = K_3$ (2) $K_2K_3 = K_1$

(3) $K_3 = K_1K_2$ (4) $K_3 \cdot K_2^3 = K_1^2$

**Answer (3)**

**Hints :** $CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g)$

$$K_1 = \frac{[CO_2][H_2]}{[CO][H_2O]} \quad ....(i)$$

$CH_4(g) + H_2O(g) \rightleftharpoons CO(g) + 3H_2(g)$

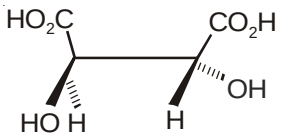
$$K_2 = \frac{[CO][H_2]^3}{[CH_4][H_2O]} \quad ...(ii)$$

$CH_4(g) + 2H_2O(g) \rightleftharpoons CO_2(g) + 4H_2(g)$

$$K_3 = \frac{[CO_2][H_2]^4}{[CH_4][H_2O]^2} \quad ...(iii)$$

on equating (i), (ii) and (iii), we get $K_3 = K_1 K_2$

51. The absolute configuration of




is

(1) S, S (2) R, R

(3) R, S (4) S, R

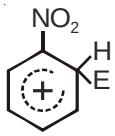
**Answer (2)**

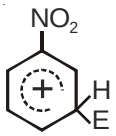
**Hints :** Absolute configuration is R, R

52. The electrophile, $E^{\oplus}$ attacks the benzene ring to generate the intermediate σ-complex. Of the following which σ-complex is of lowest energy?

(1) 

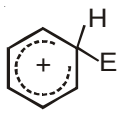

(2) 


(3) 


(4) 


**Answer (2)**

**Hints :**




This intermediate σ-complex is most stable than rest of three intermediate σ-complex. Therefore it has lowest energy.

53. α-D-(+)-glucose and β-D-(+)-glucose are
(1) Conformers (2) Epimers
(3) Anomers (4) Enantiomers

**Answer (3)**

**Hints :**

α - D (+) - glucose and β – D (+) - glucose have different configuration about C – 1 carbon therefore these are anomers.

54. Standard entropy of $X_2$, $Y_2$ and $XY_3$ are 60, 40 and 50 $JK^{-1}$ $mol^{-1}$, respectively. For the reaction, $\frac{1}{2}X_2 + \frac{3}{2}Y_2 \rightarrow XY_3$, $\Delta H = -30$ kJ, to be at equilibrium, the temperature will be
(1) 1250 K (2) 500 K
(3) 750 K (4) 1000 K

**Answer (3)**

**Hints :**

$$\frac{1}{2}X_2 + \frac{3}{2}Y_2 \longrightarrow XY_3$$

$$\Delta S_{reaction} = 50 - \frac{3}{2}\times 40 - \frac{1}{2}\times 60$$

$$= -40 \text{ J } K^{-1} \text{ mol}^{-1}$$

$\Delta G = \Delta H - T\Delta S$ at equilibrium $\Delta G = 0$

$\Delta H = T\Delta S$

$$T = \frac{-30\times 1000}{-40} = 750 \text{ K}$$

55. Four species are listed below :
i. $HCO_3^-$ ii. $H_3O^+$
iii. $HSO_4^-$ iv. $HSO_3F$
(1) iv < ii < iii < i (2) ii < iii < i < iv
(3) i < iii < ii < iv (4) iii < i < iv < ii

**Answer (3)**

**Hints :**

The correct sequence of acidic strength

$$HCO_3^- < HSO_4^- < H_3O^+ < HSO_3F$$

56. Which one of the following constitutes a group of the isoelectronic species?
(1) $C_2^{2-}$, $O_2^-$, CO, NO (2) $NO^+$, $C_2^{2-}$, $CN^-$, $N_2$
(3) $CN^-$, $N_2$, $O_2^{2-}$, $C_2^{2-}$ (4) $N_2$, $O_2^-$, $NO^-$, CO

**Answer (2)**

**Hints :** $NO^+, C_2^{2-}, CN^-$ and $N_2$ all are having 14 electrons therefore these are isoelectronic species.

57. Which one of the following pairs of species have the same bond order?
(1) $CN^-$ and $NO^+$ (2) $CN^-$ and $CN^+$
(3) $O_2^-$ and $CN^-$ (4) $NO^+$ and $CN^+$

**Answer (1)**

**Hints :** $CN^-$ and $NO^+$ both have 14 electrons therefore bond order will be same.

58. The ionization enthalpy of hydrogen atom is $1.312 \times 10^6$ J mol$^{-1}$. The energy required to excite the electron in the atom from n = 1 to n = 2 is
(1) $8.51 \times 10^5$ J mol$^{-1}$ (2) $6.56 \times 10^5$ J mol$^{-1}$
(3) $7.56 \times 10^5$ J mol$^{-1}$ (4) $9.84 \times 10^5$ J mol$^{-1}$

**Answer (4)**

**Hints :**

$$E_n = -1.312 \times 10^6 \, z^2/n^2 \, J/mol$$

$$\Delta E = E_2 - E_1 = 1.312 \times 10^6 \left[1 - \frac{1}{4}\right]$$

$$= 1.312 \times 10^6 \left(\frac{3}{4}\right)$$

$$= 9.84 \times 10^5 \, Jmol^{-1}$$

59. Which one of the following is the correct statement?
(1) Boric acid is a protonic acid
(2) Beryllium exhibits coordination number of six
(3) Chlorides of both beryllium and aluminium have bridged chloride structures in solid phase
(4) $B_2H_6.2NH_3$ is known as 'inorganic benzene'

**Answer (3)**

**Hints :**

Chlorides of Beryllium and aluminium have chlorobridged structure in solid phase

60. Given $E^o_{Cr^{3+}/Cr} = -0.72$ V, $E^o_{Fe^{2+}/Fe} = -0.42$ V. The potential for the cell
Cr | $Cr^{3+}$ (0.1 M) | $Fe^{2+}$ (0.01 M) | Fe is
(1) 0.26 V (2) 0.339 V
(3) –0.339 V (4) –0.26 V

**Answer (1)**

**Hints :** $E^0_{cell} = E^0_{RP\ cathode} - E^0_{RP\ anode}$

= –0.42 – (–0.72)

= 0.30 V

$$Cr \longrightarrow Cr^{3+} + 3e$$

$$Fe^{2+} + 2e \longrightarrow Fe$$

net cell reaction $2Cr_{(s)} + 3Fe^{2+}_{(aq.)} \longrightarrow 2Cr^{3+}_{(aq.)} + 3Fe(s)$

$$E_{cell} = E^0_{cell} - \frac{0.059}{6} \log \frac{[Cr^{3+}]^2}{[Fe^{2+}]^3}$$

$$= 0.30 - \frac{0.059}{6} \times \log \frac{(0.1)^2}{(0.01)^3}$$

= 0.30 – 0.04

= 0.26 V

61. Amount of oxalic acid present in a solution can be determined by its titration with $KMnO_4$ solution in the presence of $H_2SO_4$. The titration gives unsatisfactory result when carried out in the presence of HCl, because HCl

(1) Gets oxidised by oxalic acid to chlorine

(2) Furnishes $H^-$ ions in addition to those from oxalic acid

(3) Reduces permanganate to $Mn^{2+}$

(4) Oxidises oxalic acid to carbon dioxide and water

**Answer (3)**

**Hints :** $2KMnO_4 + 16HCl \longrightarrow 2KCl + MnCl_2 + 5Cl_2 + 8H_2O$

$\therefore$ $KMnO_4$ reduces to $Mn^{2+}$ by HCl

62. The vapour pressure of water at 20ºC is f17.5 mm Hg. If 18 g of glucose ($C_6H_{12}O_6$) is added to 178.2 g of water at 20ºC, the vapour pressure of the resulting solution will be

(1) 17.675 mm Hg (2) 15.750 mm Hg

(3) 16.500 mm Hg (4) 17.325 mm Hg

**Answer (4)**

**Hints :**

$$\frac{P^\circ - P_s}{P_s} = \frac{n}{N}$$

$$\frac{17.5 - P_s}{P_s} = \frac{18/180}{\frac{178.2}{18}}$$

$$\frac{17.5 - P_s}{P_s} = \frac{0.1}{9.9}$$

$\therefore$ $P_s = 17.325$ mm Hg

63. Among the following substituted silanes the one which will give rise to cross linked silicone polymer on hydrolysis is

(1) $R_4Si$ (2) $RSiCl_3$

(3) $R_2SiCl_2$ (4) $R_3SiCl$

**Answer (2)**

**Hints :**

$RSiCl_3$ on hydrolysis forms cross linked silicones

64. In context with the industrial preparation of hydrogen from water gas ($CO+H_2$), which of the following is the correct statement?

(1) CO and $H_2$ are fractionally separated using differences in their densities

(2) CO is removed by absorption in aqueous $Cu_2Cl_2$ solution

(3) $H_2$ is removed through occlusion with Pd

(4) CO is oxidised to $CO_2$ with steam in the presence of a catalyst followed by absorption of $CO_2$ in alkali

**Answer (4)**

**Hints :**

$$\underset{\text{(red hot)}}{C} + \underset{\text{(steam)}}{H_2O} \longrightarrow \underbrace{CO + H_2}_{\text{water gas}}$$

65. In a compound, atoms of element Y form ccp lattice and those of element X occupy 2/3rd of tetrahedral voids. The formula of the compound will be

(1) $X_4Y_3$ (2) $X_2Y_3$

(3) $X_2Y$ (4) $X_3Y_4$

**Answer (1)**

**Hints :** Effective number of atoms of Y = 4

effective numebr of atoms of X = $\frac{2}{3} \times 8$

$\therefore$ formula of compound $X_4 Y_3$

66. Gold numbers of protective colloids A, B, C and D are 0.50, 0.01, 0.10 and 0.005, respectively. The correct order of their protective powers is

(1) D < A < C <B (2) C < B < D < A

(3) A < C < B < D (4) B < D < A < C

**Answer (3)**

**Hints :**

Lesser value of gold number, more will be protective power therefore

D > B > C > A

67. The hydrocarbon which can react with sodium in liquid ammonia is

(1) $CH_3CH_2CH_2C \equiv CCH_2CH_2CH_3$ (2) $CH_3CH_2C \equiv CH$

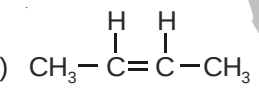
(3) $CH_3CH = CHCH_3$ (4) $CH_3CH_2C \equiv CCH_2CH_3$

**Answer (2)**

**Hints :** $CH_3 - CH_2 - C \equiv CH$ has acidic hydrogen therefore reacts with Na in liquid $NH_3$

68. The treatment of $CH_3MgX$ with $CH_3C \equiv C - H$ produces

(1) $CH_3 - CH = CH_2$ (2) $CH_3C \equiv C - CH_3$

(3)  (4) $CH_4$

**Answer (4)**

**Hints :** $CH_3 - Mgx + CH_3 - C \equiv C - H \longrightarrow CH_4 + CH_3 - C \equiv C - Mgx$

69. The correct decreasing order of priority for the functional groups of organic compounds in the IUPAC system of nomenclature is

(1) $-COOH, -SO_3H, -CONH_2, -CHO$ (2) $-SO_3H, -COOH, -CONH_2, -CHO$

(3) $-CHO, -COOH, -SO_3H, -CONH_2$ (4) $-CONH_2, -CHO, -SO_3H, -COOH$

**Answer (1)**

**Hints :**

On the basis of priority rule.

70. The $pK_a$ of a weak acid, HA, is 4.80. The $pK_b$ of a weak base, BOH, is 4.78. The pH of an aqueous solution of the corresponding salt, BA, will be

(1) 9.58 (2) 4.79

(3) 7.01 (4) 9.22

**Answer (3)**

**Hints :**

$$pH = 7 + \frac{1}{2}pK_a - \frac{1}{2}pK_b$$

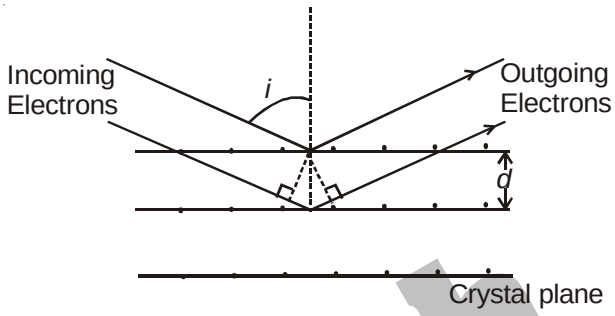
$$= 7 + \frac{1}{2} \times 4.80 - \frac{1}{2} \times 4.78$$

$$= 7.01$$

# PART - C
# PHYSICS

***Directions :*** *Question No. 71, 72 and 73 are based on the following paragraph.*

Wave property of electrons implies that they will show diffraction effects. Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively (see figure).




71. Electrons accelerated by potential $V$ are diffracted from a crystal. If $d$ = 1 A and $i = 30°$. $V$ should be about $h = 6.6 \times 10^{-34}$ Js, $m_e = 9.1 \times 10^{-31}$ kg. $e = 1.6 \times 10^{-19}$ C

(1) 2000 V (2) 50 V

(3) 500 V (4) 1000 V

**Answer (2)**

**Hints :** $\Delta x = 2d\cos 30° = n\lambda$

$$d\sqrt{3} = n\lambda$$

$$\Rightarrow \quad \lambda = \frac{d\sqrt{3}}{n}$$

as $d$ = 1 Å

$$\lambda \approx \sqrt{3} = \sqrt{\frac{150}{V}}$$

$$\Rightarrow \quad V = 50 \text{ V}$$

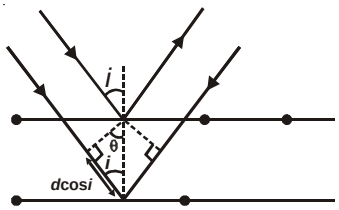
72. If a strong diffraction peak is observed when electrons are incident at an angle '$i$' from the normal to the crystal planes with distance '$d$' between them (see figure), de Broglie wavelength $\lambda_{dB}$ of electrons can be calculated by the relationship ($n$ is an integer)

(1) $d \sin i = n \lambda_{dB}$ (2) $2d \cos i = n \lambda_{dB}$

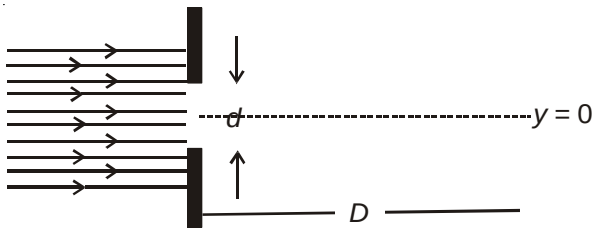
(3) $2d \sin i = n \lambda_{dB}$ (4) $d \cos i = n \lambda_{dB}$

**Answer (2)**

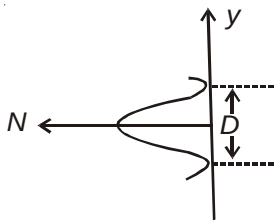
**Hints :** $\Delta x = 2d\sin\theta = 2d\cos i$

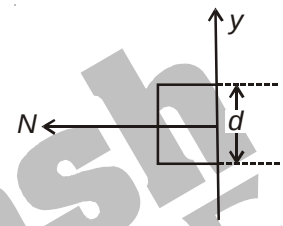


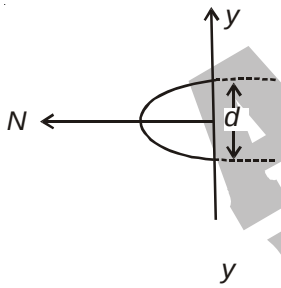
73. In an experiment, electrons are made to pass through a narrow slit of width '*d*' comparable to their de Broglie wavelength. They are detected on a screen at a distance '*D*' from the slit (see figure).

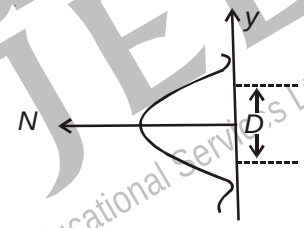



Which of the following graphs can be expected to represent the number of electrons '*N*' detected as a function of the detector position '*y*' (*y* = 0 corresponds to the middle of the slit)?

(1) 


(2) 


(3) 


(4) 


**Answer (4)**

**Hints :** The diffraction effect causes the beam to floor

74. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km $s^{-1}$, the escape velocity from the surface of the planet would be

(1) 1.1 km $s^{-1}$

(2) 11 km $s^{-1}$

(3) 110 km $s^{-1}$

(4) 0.11 km $s^{-1}$

**Answer (3)**

**Hints :** $v_e = \sqrt{\frac{GM_e}{R_e}} = 11$ km/s

$M_p = 10\, M_e$ $\qquad R_p = \frac{R_e}{10}$

$v_p = \sqrt{\frac{G10M_e}{\frac{R_e}{10}}} = 10\sqrt{\frac{GM_e}{R_e}} = 110$ km/s

75. A spherical solid ball of volume $V$ is made of a material of density $\rho_1$. It is falling through a liquid of density $\rho_2$ ($\rho_2 < \rho_1$). Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed $v$. i.e., $F_{viscous} = -kr^2$, $k > 0$. The terminal speed of the ball is

(1) $\sqrt{\frac{Vg(\rho_1-\rho_2)}{k}}$ (2) $\frac{Vg\rho_1}{k}$

(3) $\sqrt{\frac{Vg\rho_1}{k}}$ (4) $\frac{Vg(\rho_1-\rho_2)}{k}$

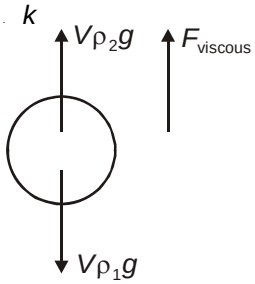
**Answer (1)**

**Hints :** For terminal velocity

$V\rho_2 g = F_{viscous} + V\rho_1 g$

$\Rightarrow kV^2 = V(\rho_1 - \rho_2)g$

$\Rightarrow \quad V = \sqrt{\frac{V(\rho_1-\rho_2)g}{k}}$




76. Shown in the figure below is a meter-bridge set up with null deflection in the galvanometer.

The value of the unknown resistor $R$ is

(1) 13.75 Ω (2) 220 Ω

(3) 110 Ω (4) 55 Ω

**Answer (2)**

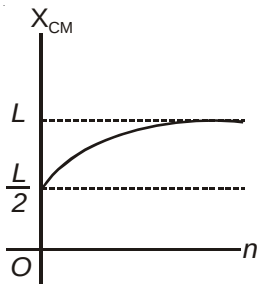
**Hints :** For balanced meter bridge

$\Rightarrow \quad \frac{P}{l} = \frac{R}{100-l}$

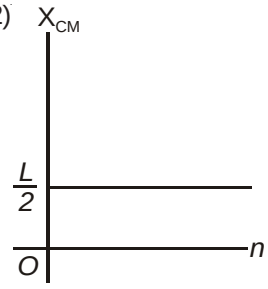
$\Rightarrow \quad \frac{55}{20} = \frac{R}{80} \Rightarrow R = 220\ \Omega$

77. A thin rod of length '$L$' is lying along the $x$-axis with its ends at $x = 0$ and $x = L$. Its linear density (mass/length) varies with $x$ and $k\left(\frac{x}{L}\right)^n$. Where $n$ can be zero or any positive number. If the position $X_{CM}$ of the centre of mass of the rod is plotted against '$n$'. Which of the following graphs best approximates the dependence of $X_{CM}$ on n?

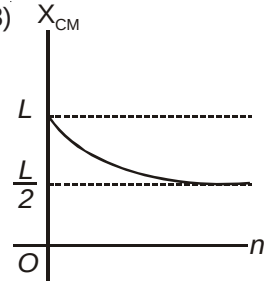
(1)



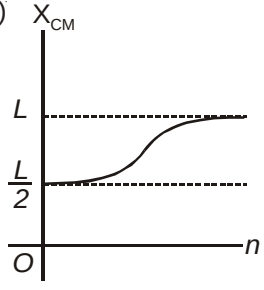

(2)




(3)

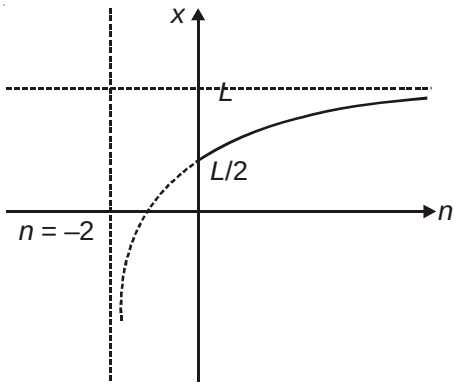



(4)



**Answer (1)**

**Hints :** $x_{cm} = \dfrac{\int_0^L \lambda x.dx}{\int_0^2 dx} = \dfrac{\int_0^L k\dfrac{x^{n+1}}{L^n}dx}{\int_0^L k\dfrac{x^n}{L^n}dx} = \dfrac{\left[\dfrac{x^{n+2}}{n+2}\right]_0^L}{\left[\dfrac{x^{n+1}}{n+1}\right]_0^L}$




$$= \frac{\dfrac{L^{n+2}}{n+2}}{\dfrac{L^{n+1}}{n+1}} = L\left(\frac{n+1}{n+2}\right) = L\left(1-\frac{1}{n+2}\right)$$

$$x_{cm} = \left(1-\frac{1}{n+L}\right)L$$

78. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer. She measures the column length to be x cm for the second resonance. Then

(1) $18 > x$ (2) $x > 54$

(3) $54 > x > 36$ (4) $36 > x > 18$

**Answer (2)**

**Hints :** First resonance

$$l_1\frac{\lambda}{4} = 18\text{ cm}$$

Second resonance must be at

$$l_2 = \frac{3\lambda}{4} = 54\text{ cm}$$

In summer, the speed of sound in air increases so wavelength also increases. The second resonance will be more than 54 cm.

79. The dimension of magnetic field in M, L, T and C (Coulomb) is given as

(1) $MLT^{-1}C^{-1}$ (2) $MT^2C^{-2}$

(3) $MT^{-1}C^{-1}$ (4) $MT^{-2}C^{-1}$

**Answer (3)**

**Hints :** $F = q\,v\,B$

$\Rightarrow$ $[MLT^{-2}] = [CLT^{-1}]$ [dimension of $B$]

$\Rightarrow$ $[B] = MC^{-1}T^{-1}$

80. Consider a uniform square plate of side '$a$' and mass '$m$'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

(1) $\frac{5}{6}ma^2$ (2) $\frac{1}{12}ma^2$

(3) $\frac{7}{12}ma^2$ (4) $\frac{2}{3}ma^2$

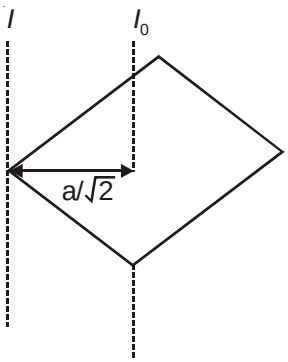
**Answer (4)**

**Hints :** $I = I_0 + Md^2$

$$= M\frac{a^2}{6} + M\left(\frac{a}{\sqrt{2}}\right)^2$$

$$= \frac{Ma^2}{6} + \frac{Ma^2}{2}$$

$$= \frac{2}{3}Ma^2$$




81. A body of mass $m$ = 3.513 kg is moving along the $x$-axis with a speed of 5.00 ms$^{-1}$. The magnitude of its momentum is recorded as

(1) 17.6 kg ms$^{-1}$ (2) 17.565 kg ms$^{-1}$

(3) 17.56 kg ms$^{-1}$ (4) 17.57 kg ms$^{-1}$

**Answer (1)**

**Hints :** $p = mv$

= (3.513) × (5.00)

= 17.565

The answer should be reported up to 3 significant figures.

82. An athlete in the Olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range

(1) 200 J – 500 J (2) $2 \times 10^5$ J – $3 \times 10^5$ J

(3) 20,000 J – 50,000 J (4) 2,000 J – 5,000 J

**Answer (4)**

**Hints :** K.E. $= \frac{1}{2}mv^2$

mass can be approx. 60 kg or in between 50 kg to 100 kg

$v$ can be 10 m/s (average)

$\therefore$ Answer is 2000 J to 5000 J

83. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is '$d$'. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $k_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $k_2 = 6$ and thickness $\frac{2d}{3}$. Capacitance of the capacitor is now

(1) 1.8 pF (2) 45 pF

(3) 40.5 pF (4) 20.25 pF

**Answer (3)**

**Hints :** Air cored capacitor's capacitance is

$$\frac{\varepsilon_0 A}{d} = 9\text{pF}$$

When the capacitor is filled with the dielectrics

$$C' = \frac{\varepsilon_0 A}{\frac{t_1}{K_1} + \frac{t_1}{K_1}} = \frac{\varepsilon_0 A}{\frac{d}{9} + \frac{2d}{18}}$$

$$= 9\frac{\varepsilon_0 A}{2d} = 40.5 \text{ pF}$$

84. The speed of sound in oxygen ($O_2$) at a certain temperature is 460 $ms^{-1}$. The speed of sound in helium (He) at the same temperature will be assume both gases to be ideal)

(1) 460 $ms^{-1}$ (2) 500 $ms^{-1}$

(3) 650 $ms^{-1}$ (4) 330 $ms^{-1}$

**Answer (No Choice is Correct)**

**Hints :** $460 = \sqrt{\frac{7/5 RT}{32\times10^{-3}}}$

$$V = \sqrt{\frac{5/3 RT}{4\times10^{-3}}}$$

$$\Rightarrow \frac{460}{V} = \sqrt{\frac{21}{25\times8}}$$

$$\Rightarrow V \approx 460\sqrt{10} \text{ m/s}$$

85. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement -1 : Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion.

and

Statement-2 : For heavy nuclei, binding energy per nucleon increases with increasing $Z$ wihile for light nuclei it decreases with increasing $Z$.

(1) Statement-1 is false, Statement-2 is true

(2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(3) Statement-1 is ture, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

(4) Statement-1 is ture, Statement-2 is false

**Answer (4)**

**Hints :** *B.E.* per nucleon for middle sized nuclei is maximum.

86. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement -1 : For a mass $M$ kept at the centre of a cube of side '$a$', the flux of gravitational field passing through its sides is $4\pi GM$.

and

Statement-2 : If the direction of field due to a point source is radial and its dependence on the distance '$r$' from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

(1) Statement-1 is false, Statement-2 is true

(2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(3) Statement-1 is ture, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

(4) Statement-1 is ture, Statement-2 is false

**Answer (2)**

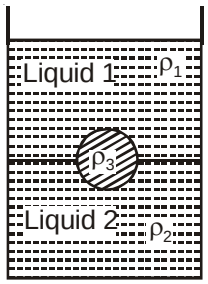
**Hints :** $\frac{1}{4\pi\varepsilon_0}$ is equivalent to $G$ in electrostatics

$\phi_{Gravitational} = 4\pi G$ [source of gravitational field]

$= 4\pi GM$

87. A jar is filled with two non-mixing liquid 1 and 2 having densities $\rho_1$ and $\rho_2$ respectively. A solid ball, made of a material of density $\rho_3$, is dropped in the jar. It comes to equilibrium in the position shown in the figure.

Which of the following is true for $\rho_1, \rho_2$ and $\rho_3$?




(1) $\rho_3 < \rho_1 < \rho_2$ (2) $\rho_1 > \rho_3 > \rho_2$

(3) $\rho_1 < \rho_2 < \rho_3$ (4) $\rho_1 < \rho_3 < \rho_2$

**Answer (4)**

**Hints :** $\rho_3 = \frac{\rho_1 + \rho_2}{2}$ and $\rho_1 < \rho_2$

So $\rho_1 < \rho_3 < \rho_2$

88. A working transistor with its three legs marked *P*, *Q* and *R* is tested using a multimeter. No conduction is found between *P* and *Q*. By connecting the common (negative) terminal of the multimeter or *R* and the other (positive) terminal to *P* or *Q*, some resistance is seen on the multimeter. Which of the following is true for the transistor?

(1) It is a *npn* transistor with *R* as base (2) It is a *pnp* transistor with *R* as collector

(3) It is a *pnp* transistor with *R* as emitter (4) It is a *npn* transitor with *R* as collector

**Answer (1)**

**Hints :** If we place negative plate of multimeter at any pin and positive lead at any other of two pins and we get same voltage, then pin with negative voltage is base.

**Directions : Questions No. 89 and 90 are based on the following paragraph.**

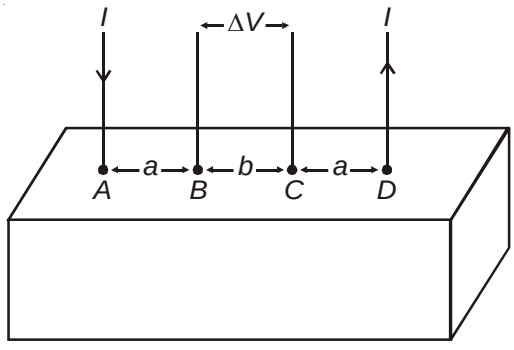
Consider a block of conducting material of resistivity '$\rho$' shown in the figure. Current *I* enters at *A* and leaves from *D*. We apply superposition principle to find voltage $\Delta V$ developed between *B* and *C*. The calculation is done in the following steps :

(i) Take current *I* entering from *A* and assume it to spread over a hemispherical surface in the block.

(ii) Calculate field *E*(*r*) at distance '*r*' from *A* by using Ohm's law $E = \rho j$, where *j* is the current per unit area at '*r*'.

(iii) From the '*r*' dependence of *E*(*r*), obtained the potential *V*(*r*) at *r*.

(iv) Repeat (i), (ii), and (iii) for current *I* leaving *D* and superpose results for *A* and *D*.



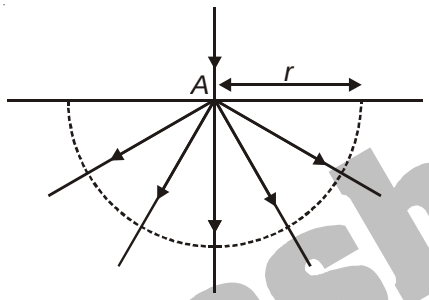
89. $\Delta V$ measured between $B$ and $C$ is

(1) $\frac{\rho l}{\pi a} - \frac{\rho l}{\pi(a+b)}$ (2) $\frac{\rho l}{a} - \frac{\rho l}{(a+b)}$

(3) $\frac{\rho l}{2\pi a} - \frac{\rho l}{2\pi(a+b)}$ (4) $\frac{\rho l}{2\pi(a-b)}$

**Answer (1)**

**Hints :** For current entering at $A$




Current density at a distance $r = \frac{I}{2\pi r^2}$

Electric field $E = \rho j$

$= \frac{\rho l}{2\pi r^2}$

$V_B - V_C = \int_a^{a+b} \frac{\rho l}{2\pi r^2}$

$= \frac{\rho l}{2\pi a} - \frac{\rho l}{2\pi(a+b)}$

Same potential difference will be due to current coming out. So, net P.D.

$\Delta V = 2\left(\frac{\rho l}{2\pi a} - \frac{\rho l}{2\pi(a+b)}\right)$

$= \frac{\rho l}{\pi a} - \frac{\rho l}{\pi(a+b)}$

90. For current entering at $A$, the electric field at distance '$r$' from $A$ is

(1) $\frac{\rho l}{8\pi r^2}$ (2) $\frac{\rho l}{r^2}$

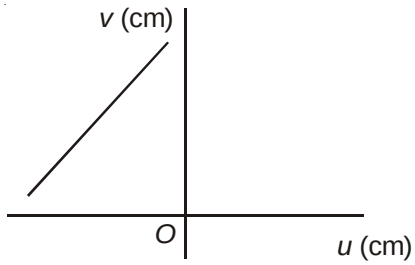
(3) $\frac{\rho l}{2\pi r^2}$ (4) $\frac{\rho l}{4\pi r^2}$

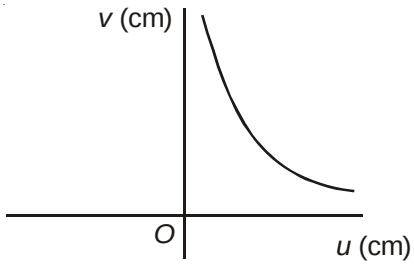
**Answer (3)**

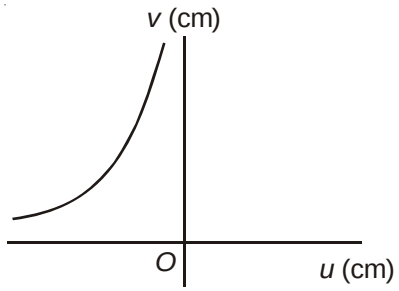
**Hints :** Electric field $E = \rho j$

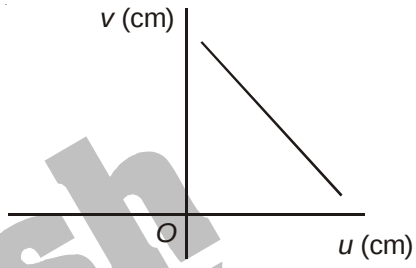
$= \frac{\rho l}{2\pi r^2}$

91. A student measures the focal length of a convex lens by putting an object pin at a distance *u* from the lens and measuring the distance *v* of the image pin. The graph between *u* and *v* plotted by the student should look like

(1) 


(2) 


(3) 


(4) 


**Answer (3)**

**Hints :** For $u = -\infty$ $v = f$

and for $u = -f$ $v = \infty$

Best possible graph is third.

92. A block of mass 0.50 kg is moving with a speed of 2.00 $ms^{-1}$ on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is

(1) 0.16 J (2) 1.00 J

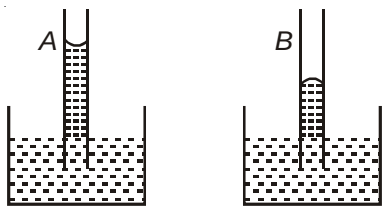
(3) 0.67 J (4) 0.34 J

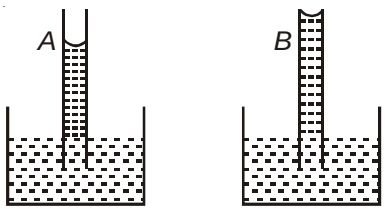
**Answer (3)**

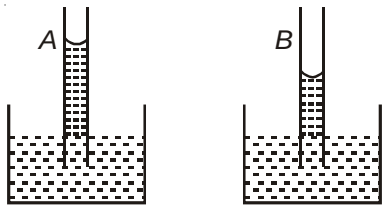
**Hints :** $\Delta K = \frac{1}{2}\frac{m_1 m_2}{(m_1+m)}(v_1 - v_2)^2$

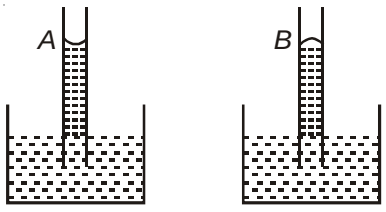
$= \frac{1}{2}\frac{0.5}{1.5} \times (2)^2 = \frac{2}{3}\text{ J} = 0.67\text{ J}$

93. A capillary tube (*A*) is dipped in water. Another identical tube (*B*) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes?

(1) 


(2) 


(3) 


(4) 

**Answer (3)**

**Hints :** On adding soap surface tension of water decreases. So, less capillary rise will take place.

94. Suppose an electron is attracted towards the origin by a force $\frac{k}{r}$ where $k$ is a constant and $r$ is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the $n^{th}$ orbital of the electron is found to be $r_n$ and the kinetic energy of the electron to be $T_n$. Then which of the following is true?

(1) $T_n \propto \frac{1}{n^2} \cdot r_n \propto n^2$ (2) $T_n$ independent of $n \cdot r_n \propto n$

(3) $T_n \propto \frac{1}{n} \cdot r_n \propto n$ (4) $T_n \propto \frac{1}{n} \cdot r_n \propto n^2$

**Answer (2)**

**Hints :** $F_e = \frac{mv^2}{r}$

$\frac{k}{r} = \frac{mv^2}{r} \Rightarrow mv^2 = k$

$\Rightarrow$ kinetic energy = $\frac{k}{2}$

So kinetic energy is independent $n$.

Since $mvr = \frac{nh}{2\pi}$

and $mv^2 = k$

so $r \propto n$

95. A wave travelling along the x-axis is described by the equation $y(x, t) = 0.005 \cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 and 2.0 s. respectively, then $\alpha$ and $\beta$ in appropriate units are

(1) $\alpha = 25.00\ \pi, \beta = \pi$ (2) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$

(3) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$ (4) $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$

**Answer (1)**

**Hints** :

$\alpha = \frac{2\pi}{\lambda} = \frac{2\pi}{0.08} = 25.00\ \pi$

$\beta = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

96. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A = 10\ cm^2$ and length = 20 cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is ($\mu_0 = 4\ \pi \times 10^{-7}$ T m $A^{-1}$)

(1) $2.4\ \pi \times 10^{-5}$ H (2) $4.8\ \pi \times 10^{-4}$ H

(3) $4.8\ \pi \times 10^{-5}$ H (4) $2.4\ \pi \times 10^{-4}$ H

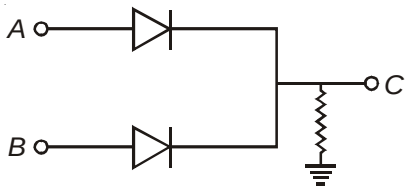
**Answer (4)**

**Hints :**

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \frac{4\pi \times 10^{-7} \times 300 \times 400 \times 10^{-3}}{0.2}$$

$= 2.4\ \pi \times 10^{-4}$ H

97. In the circuit below, *A* and *B* represent two inputs and *C* represents the output.




The circuit represents

(1) NOR gate (2) AND gate

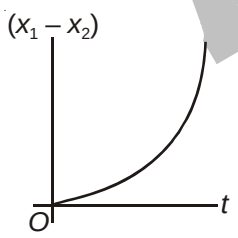
(3) NAND gate (4) OR gate

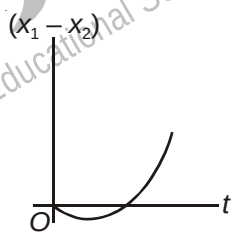
**Answer (4)**

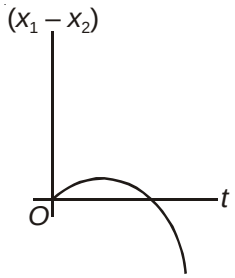
**Hints :**

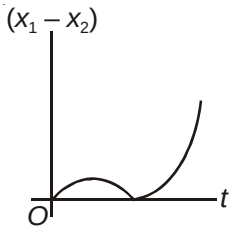
When either *A* or *B* is at high potential, output at *C* will be high

98. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive $x$-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive $x$-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time $t$ and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time $t$?

(1) 


(2) 


(3) 


(4) 


**Answer (2)**

**Hints :**

$$x_1 = \frac{1}{2} a t^2$$

$x_2 = ut$

$$x_1 - x_2 = \frac{1}{2} a t^2 - ut$$

This is a parabola, cancave up

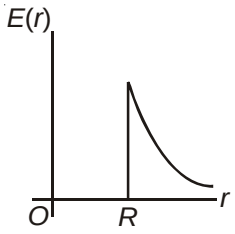
99. An experiment is performed to find the refractive index of glass using a travelling microscope. In this experiment distance are measured by

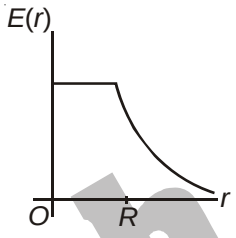
(1) A vernier scale provided on the microscope
(2) A standard laboratory scale
(3) A meter scale provided on the microscope
(4) A screw gauge provided on the microscope

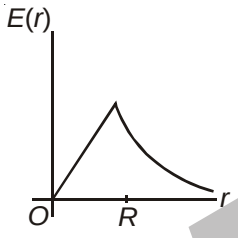
**Answer (1)**

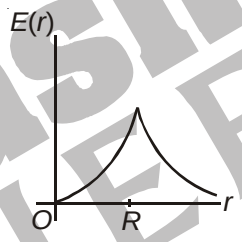
**Hints :** A vernier scale is attached with the travelling microscope.

100. A thin spherical shell of radius $R$ has charge $Q$ spread uniformly over its surface. Which of the following graphs most closely represents the electric field $E(r)$ produced by the shell in the range $0 \le r < \infty$, where $r$ is the distance from the centre of the shell?

(1) 


(2) 


(3) 


(4) 

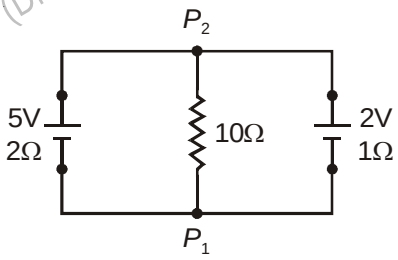

**Answer (1)**

**Hints :**

for $r < R$, $E = 0$

for $r \ge R$, $E = \left(\frac{1}{4\pi\varepsilon}\right)\frac{q}{r^2}$

101. A 5 V battery with internal resistance 2 Ω and a 2 V battery with internal resistance 1 Ω are connected to a 10 Ω resistor as shown in the figure.




The current in the 10 Ω resistor is

(1) 0.27 A $P_2$ to $P_1$
(2) 0.03 A $P_1$ to $P_2$
(3) 0.03 A $P_2$ to $P_1$
(4) 0.27 A $P_1$ to $P_2$

**Answer (3)**

**Hints :** P. D. between $p_2$ and $p_1$ is

$$V = \frac{\frac{5}{2} - \frac{2}{1} + \frac{0}{10}}{\frac{1}{2} + \frac{1}{1} + \frac{1}{10}} = \frac{5}{16}$$

$$i = \frac{V}{10} = \frac{5}{160}\text{A}$$

102. A horizontal overhead powerline is at a height of 4 m from the ground and carries a current of 100 A from east to west. the magnetic field directly below it on the ground is ($\mu_0 = 4\pi \times 10^{-7}$ T m $A^{-1}$)

(1) $2.5 \times 10^{-7}$ T southward

(2) $5 \times 10^{-6}$ T northward

(3) $5 \times 10^{-6}$ 4T southward

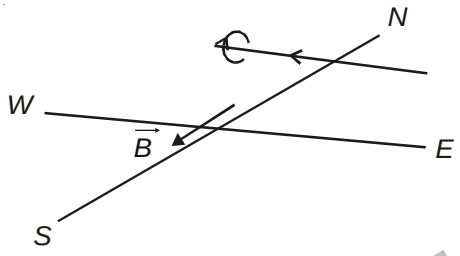
(4) $2.5 \times 10^{-7}$ T northward

**Answer (3)**

**Hints :**

$$B = \frac{\mu_0 i}{2\pi r} = 5 \times 10^{-6} \text{ T}$$

By thumb rule, field is due south




103. Relative permittivity and permeability of a material are $\varepsilon_r$ and $\mu_r$, respectively. Which of the following values of these quantities are allowed for a diamagnetic material?

(1) $\varepsilon_r = 0.5$, $\mu_r = 1.5$

(2) $\varepsilon_r = 1.5$, $\mu_r = 0.5$

(3) $\varepsilon_r = 0.5$, $\mu_r = 0.5$

(4) $\varepsilon_r = 1.5$, $\mu_r = 1.5$

**Answer (2)**

**Hints :**

$\varepsilon_r \geq 1$ for any material

$\mu_r \leq 1$ for diamagnetics

104. Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of –0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is

(1) 3.32 mm

(2) 3.73 mm

(3) 3.67 mm

(4) 3.38 mm

**Answer (4)**

**Hints :**

$$\text{least count} = \frac{\text{pitch}}{\text{CSD}} = \frac{1\text{mm}/2}{50} = 0.01 \text{ mm}$$

Reading = 3 mm + 35 × 0.01 = 3.35 mm

Actual reading = 3.35 – (– 0.03)

= 3.38 mm

105. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume $V_1$ and contains ideal gas at pressure $P_1$ and temperature $T_1$. The other chamber has volume $V_2$ and contains ideal gas at pressure $P_2$ and temperature $T_2$. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be

(1) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_2+P_2V_2T_1}$ (2) $\frac{P_1V_1T_1+P_2V_2T_2}{P_1V_1+P_2V_2}$

(3) $\frac{P_1V_1T_2+P_2V_2T_1}{P_1V_1+P_2V_2}$ (4) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_1+P_2V_2T_2}$

**Answer (1)**

**Hints :**

**First method**

$n_1 + n_2 = n$

$$\Rightarrow \frac{P_1V_1}{T_1}+\frac{P_2V_2}{T_2}=\frac{P(V_1+V_2)}{T} \quad ...(1)$$

Also, $V_1 + V_2 = V$

$$\Rightarrow \frac{f}{2}P_1V_1+\frac{f}{2}P_2V_2=\frac{f}{2}PV \quad ...(2)$$

**Second method**

Loss of heat = gain of heat

$n_1C(T_1 - T) = n_2C(T - T_2)$

$$\left(\frac{P_1V_1}{RT_1}\right)R(T_1-T)=\left(\frac{P_2V_2}{RT_2}\right)(T-T_2)$$

$$\Rightarrow \quad T=\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_2+P_2V_2T_1}$$

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