CODE: D5

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HINTS & SOLUTIONS FOR AIEEE-2008

MATHS

1. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b?

(1) a = 1, b = 6 (2) a = 3, b = 4

(3) a = 0, b = 7 (4) a = 5, b = 2

Solution : (2)

$$\text{Mean} = 6 = \frac{a+b+8+5+10}{5}$$

$\Rightarrow a + b = 7$ ...(1)

$$\text{and variance, } 6^2 = 6.8 = \frac{1}{n}\left(\sum x_i^2\right) - \left(\frac{1}{n}\sum x_i\right)^2$$

$\Rightarrow 6.8 = \frac{1}{5}(a^2 + b^2 + 64 + 25 + 100) - 6^2$

$\Rightarrow a^2 + b^2 = 25$ ...(2)

Here (1) and (2) are satisfied for a = 3 and b = 4

2. The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β ?

(1) $\alpha = 2, \beta = 1$ (2) $\alpha = 1, \beta = 1$

(3) $\alpha = 2, \beta = 2$ (4) $\alpha = 1, \beta = 2$

Solution : (2) $\begin{vmatrix} \alpha & 2 & \beta \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 0$

$$\lambda\vec{a} = \left(\frac{\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{2}}\right)$$

$$\lambda\left(\alpha\hat{i} + 2\hat{j} + \beta\hat{k}\right)$$

$$\alpha\lambda = \frac{1}{\sqrt{2}}$$

$$2\lambda = \sqrt{2}$$

$$\beta\lambda = \frac{1}{\sqrt{2}}$$

$\Rightarrow \alpha = \beta = 1$

3. The non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is

(1) $\frac{\pi}{2}$ (2) p

(3) 0 (4) $\frac{\pi}{4}$

Solution : (2) $\vec{a} = 8\vec{b} \Rightarrow \vec{a} \parallel \vec{b}$

and $\vec{c} = -7\vec{b} \Rightarrow \vec{b} = \frac{-\vec{c}}{7} \Rightarrow \vec{b} \parallel -\vec{c}$

$\Rightarrow \vec{a} \parallel -\vec{c}$ $\therefore$ Angle between $\vec{a}$ and $\vec{c}$ = π

4. The line passing through the points (5, 1, a) and (3, 2, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then

(1) a = 6, b = 4 (2) a = 8, b = 2

(3) a = 2, b = 8 (4) a = 4, b = 6

Solution : (1) The equation of line passing through (5, 1, a) and (3, b, 1) is

$$\frac{x-5}{5-3} = \frac{y-1}{1-b} = \frac{z-a}{a-1}$$

Which crosses y - z plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$

$\therefore$ $2r + 5 = 0 \Rightarrow r = -5/2$

and $r(1 - b) + 1 = \frac{17}{2} \Rightarrow b = 4$

and $r(a - 1) + a = -\frac{13}{2} \Rightarrow a = 6$

5. If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ intersect at a point, then the integer k is equal to

(1) 2 (2) –2

(3) –5 (4) 5

Solution : (3)

Given lines are

$\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3} = r_1$

$\Rightarrow x = k r_1 + 1, y = 2r_2 + 2, z = 3r_1 + 3$

and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2} = r_2$, say

$\therefore$ $x = 3r_2 + 2, y = kr_2 + 3, z = 2r_2 + 1$

If they intersec each other now

$kr_1 + 1 = 3r_2 + 2 \Rightarrow kr_1 - 3r_2 = 1$ ...(1)

$2r_1 + 2 = kr_2 + 3 \Rightarrow 2r_1 - kr_2 = 1$ ...(2)

$3r_1 + 3 = 2r_2 + 1 \Rightarrow 3r_1 - 2r_2 = -2$ ...(3)

Solving (1) and (2), $r_1 = \frac{k-3}{k^2-6}$ and $r_2 = \frac{2-k}{k^2-6}$

Puting values of r_1 and r_2 in (3) we get

$2k^2 + 5k - 25 = 0 \Rightarrow k = -5$

6. The differential equation of the family of circles with fixed radius 5 units and centre on the line y = 2 is

(1) $(y - 2)^2 \, y'^2 = 25 - (y - 2)^2$

(2) $(x - 2)^2 \, y'^2 = 25 - (y - 2)^2$

(3) $(x - 2)^2 \, y'^2 = 25 - (y - 2)^2$

(4) $(y - 2) \, y'^2 = 25 - (y - 2)^2$

Solution : (1)

Equation of circle is

$(x - k)^2 + (y - 2)^2 = 5^2$ ....(1)

different both solves, we get

$2(x - k) + 2(y - 2) y' = 0$

$\Rightarrow x - k = -(y - 2) y'$

$\therefore$ from (1),

$(y - 2)^2 \, y'^2 + (y - 2)^2 = 25$

$\Rightarrow (y - 2)^2 \, y'^2 = 25 - (y - 2)^2$

7. Let a. b. c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that x = cy + bz, y = az + cx, and z = bx + ay. Then $a^2 + b^2 + c^2 + 2abc$ is equal to

(1) 0 (2) 1

(3) 2 (4) –1

Solution : (2)

$\because$ $x \neq 0, y \neq 0, z \neq 0$, so gives equations has non-trivial solution

$\therefore \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix}$

$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

8. Let A be a square matrix all of whose entries are integers. Then which one of the following is true?

(1) If det $A = \pm 1$, then A^{-1} exists and all its entries are integers

(2) If det $A = \pm 1$, then A^{-1} need not exist

(3) If det $A = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers

(4) If det $A \neq \pm 1$, then A^{-1} exists and all its entries are non-integers.

Solution : (1)

$\because$ det $A \neq 0$

$\Rightarrow A^{-1}$ always exist because $|A| = \pm 1$

again cofactors of A are also integers

$\therefore$ all element of A are integers

9. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is

(1) 3 (2) 2

(3) 1 (4) 4

Solution : (2) $x^2 - 6x + a = 0$ ...(1)

$x^2 - cx + 6 = 0$ ...(2)

Let roots are $(\propto, 4k)$ & $(\propto, 3k)$

$\propto + 4k = 6$

$\propto + 3k = c$

$k = 6 - c$

$\because$ 3k is a root of (2) $\Rightarrow (3k)^2 - c(3k) + 6 = 0$

$\Rightarrow$ put $k = 6 - c$ weget $c = 5$, $c = \frac{11}{2}$

$\Rightarrow k = 1, \frac{11}{2}$

$\because$ 4 k is also a root of (I)

$\because$ put k = 1

$16 - 24 + a = 0$

$a = 8$

equal root $\propto$ can be obtained by substances (1) & (2)

$= \propto = \frac{a-6}{6-c} = \frac{8-6}{6-5} = 2$

Alternate Method

$4\alpha k = a$ ....(3)

$3\alpha k = 6$ .... (4)

$a = 8$

The root of equation (1) will be 4 and 2

Let, common root is 4

So, from equation (2), $C = \frac{11}{2}$ and the other root of the equation (2) is not an integer.

So x = 2 will be the common root.

10. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?

(1) $6 . 8 . {}^7C_4$ (2) $7 . {}^6C_4 . {}^8C_4$

(3) $8 . {}^6C_4 . {}^8C_4$ (4) $6 . 7 . {}^8C_4$

Solution : (2)

MISSISSIPPI

Arange 7 alphabets (excluding 4 s^1) in $\frac{7!}{4!2!}$

ways 4s can be inserted in 8 places in $8C_4$ ways.

$\therefore$ total different words.

$\frac{7!}{4!2!} \times {}^8C_4 = 7 \cdot {}^6C_4 \cdot {}^8C_4$

11. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$. Then which one of the following is true?

(1) $I < \frac{2}{3}$ and $J > 2$ (2) $I > \frac{2}{3}$ and $J < 2$

(3) $I > \frac{2}{3}$ and $J > 2$ (4) $I < \frac{2}{3}$ and $J < 2$

Solution : (4) $I = \int_0^1 \frac{\sin x}{\sqrt{x}}$

$J = \int_0^1 \frac{\cos x}{\sqrt{x}}$

$I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx$

$\because$ $x > \sin x$ $(x \in (0, 1))$

$\int_0^1 \frac{\sin x}{\sqrt{x}} dx < \int_0^1 \sqrt{x}\, dx$

$I < \frac{2}{3}$

Again $J = \int_0^1 \frac{\cos x}{\sqrt{x}} < \int_0^1 \frac{1}{\sqrt{x}} dx$

$J < 2$

12. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to

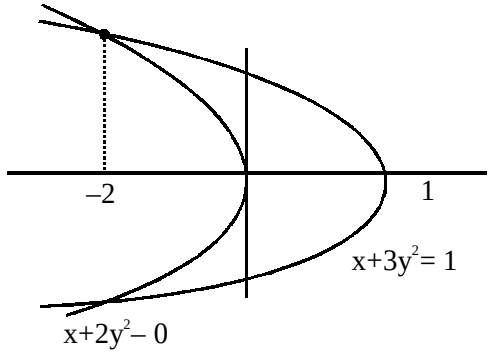
(1) $\frac{2}{3}$ (2) $\frac{4}{3}$

(3) $\frac{5}{3}$ (4) $\frac{1}{3}$

Solution : (2)

$x + 2y^2 = 0$ ...(1)

$x + 3y^2 = 1$ ...(2)



Solving (1) & (2)

Required area $= 2\left(\int_{-2}^{1}\sqrt{\frac{1-x}{3}} - \int_{-2}^{0}\sqrt{\frac{-x}{2}}\right)$

$= 2\left(\frac{1}{\sqrt{3}}\left[\frac{(1-x)^{3/2}}{-3/2}\right]_{-2}^{1} - \frac{1}{\sqrt{2}}\left[\frac{(-x)^{3/2}}{-3/2}\right]_{-2}^{0}\right)$

$= 2\left(\frac{1}{\sqrt{3}}\left[\frac{3^{3/2}}{3/2}\right] - \frac{1}{\sqrt{2}}\left[\frac{2^{3/2}}{3/2}\right]\right)$

$= 2\left(2-\frac{4}{3}\right) = \frac{4}{3}$ sq units

13. The value of $\sqrt{2}\int\frac{\sin x\,dx}{\sin\left(x-\frac{\pi}{4}\right)}$ is

(1) $x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(2) $x-\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

(3) $x+\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

(4) $x-\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

Solution : (1)

$\sqrt{2}\int\frac{\sin x\,dx}{\sin(x-\pi/4)}$

$= \sqrt{2}\int\frac{\sin(x-\pi/4+\pi/4)}{\sin(x-\pi/4)}dx$

$= \sqrt{2}\int\frac{\sin(x-\pi/4).\cos\pi/4+\cos(x-\pi/4)\sin\pi/4}{\sin(x-\pi/4)}dx$

$= \sqrt{2}\int\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}.\cot(x-\pi/4)\right)dx$

$= \int(1+\cot(x-\pi/4))dx$

$= x + \log|\sin(x-\pi/4)| + C$

14. The statement $p \to (q \to p)$ is equivalent to

(1) $p \to (p \wedge q)$ (2) $p \to (p \leftrightarrow q)$

(3) $p \to (p \to q)$ (4) $p \to (p \vee q)$

Solution : (4)

p	q	$q \to p$	$p \to (q \to p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

p	q	$p \wedge q$	$p \to (p \wedge q)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

p	q	$p \leftrightarrow q$	$p \to (p \leftrightarrow q)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	T	T

p	q	$p \to q$	$p \to (p \to q)$
T	F	T	T
T	F	F	F
T	T	T	T
F	F	T	T

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

15. The value of $\cot\left(\text{cosec}^{-1}\frac{5}{3}+\tan^{-1}\frac{2}{3}\right)$ is

(1) $\frac{4}{17}$ (2) $\frac{5}{17}$

(3) $\frac{6}{17}$ (4) $\frac{3}{17}$

Solution : (3)

$\cot\left[\sin^{-1}(3/5)+\tan^{-1}2/3\right]$

$\cot\left[\tan^{-1}(3/4)+\tan^{-1}(2/3)\right]$

$\cot\left[\tan^{-1}\frac{(3/4+2/3)}{1-\frac{3}{4}\times\frac{2}{3}}\right]$

$$\cot\left[\tan^{-1}\left(\frac{\frac{17}{12}}{\frac{12-6}{12}}\right)\right]=\frac{6}{17}$$

Directions: Questions number 16 to 20 are Assertion-Reason type questions. Each of these questions contains two statements: Statement-1 (Assertion) and Statement-2 (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

16. Let A be a 2 × 2 matrix which real entries. Let I be the 2 × 2 identity matrix. Denote by tr(A), the sum of diagonal entries of A. Assume that $A^2 = I$.

Statement-1:

If $A \neq I$ and $A \neq -I$, then det A = –1.

Statement-2:

If $A \neq I$ and $A \neq -I$, then tr (A) ≠ 0.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is false, Statement-2 is true

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Solution : (2)

$A = \begin{bmatrix} a & c \\ d & b \end{bmatrix}$, $|A| = ab - cd$

$A^2 = I$ $c(a + b) = 0$ ...(1)

$d(a + b) = 0$ ...(2)

$a^2 + cd = 1$ ...(3)

$cd + b^2 = 1$ ...(4)

from (3) & (4) $a^2 = b^2$

$a = \pm b$

if a = –b

$|A| = -a^2 - (1 - a^2)$

$= -1$

if a = b

Case - I : If a = b = 0 then cd = 1

Case - II : If $a = b \neq 0$ then $a = \pm 1$ & $b = \pm 1$

A will be I or – I otherwise |A| = –1

17. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is a rational number iff y is a transcendental number".

Statement-1:

r is equivalent to either q or p.

Statement-2:

r is equivalent to $\sim (p \leftrightarrow \sim q)$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is false, Statement-2 is true

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Solution : (3)

$\because$ $r \equiv p \Leftrightarrow q$

Now for statetement I

p	q	$p \vee q$	$r \equiv p \Leftrightarrow q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	T

for statement II

p	q	$\vee q$	$p \leftrightarrow \sim q$	$\sim (p \leftrightarrow \sim q)$	$r = p \Leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

Hence statement I is false & II is true

18. In the shop there are five types of ice-creams available. A child buys six ice-creams.

Statement-1:

The number of different ways the child can buy the six ice-creams is $^{10}C_5$.

Statement-2:

the number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is false, Statement-2 is true

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Solution : (3)

Coeffct x^6 in $(1-x)^{-5}$

$^{10}C_6$

19. ***Statement-1:***

$$\sum_{r=0}^{n}(r+1)^n C_r = (n+2)2^{n-1}.$$

Statement-2:

$$\sum_{r=0}^{n}(r+1)^n C_r x^r = (1+x)^n + nx(1+x)^{n-1}.$$

(1) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is false, Statement-2 is true

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Solution : (4) $\sum_{r=0}^{n}(r+1)^n C_r = \sum_{r=0}^{n} r\ {}^n C_r + \sum_{r=0}^{n} {}^n C_r$

$= n.2^{n-1} + 2^n = 2^{n-1}(n+2)$

20. ***Statement-1***:

For every natural number $n > 2$,

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} > \sqrt{n}.$$

Statement-2:

For every natural number $n > 2$,

$$\sqrt{n(n+1)} < n+1.$$

(1) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is false, Statement-2 is true

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Solution : (4)

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} > \left[\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n}} + \ldots + \frac{n}{\sqrt{n}}\right]$$

$$> \frac{n}{\sqrt{n}} > \sqrt{n}$$

Statement-2

$$\sqrt{n(n+1)} < n+1$$

$$\sqrt{n} < \sqrt{n+1}$$

21. The conjugate of a complex number is $\frac{1}{i-1}$. Then that complex number is

(1) $\frac{-1}{i+1}$ (2) $\frac{1}{i-1}$

(3) $\frac{-1}{i-1}$ (4) $\frac{1}{i+1}$

Solution : (1)

If $t = \frac{1}{i-1}$

$$\bar{t} = \frac{1}{-i-1}$$

22. Let R be the real line. Consider the following subsets of the plane R × R:

$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$

$T = \{(x , y) : x - y \text{ is an integer}\}$

which one of the following is true?

(1) S is an equivalence relation on R but T is not

(2) T is an equivalence relation on R but S is not

(3) Neither S nor T is an equivalence relation on R

(4) Both S and T are equivalence relations on R

Solution : (2)

Conceptual

23. Let $f : N \to Y$ be a function defined as

$f(x) = 4x + 3$ where

$Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$

So that f is invertible and its inverse is

(1) $g(y) = \frac{y+3}{4}$ (2) $g(y) = \frac{y-3}{4}$

(3) $g(y) = \frac{3y+4}{3}$ (4) $g(y) = 4 + \frac{y+3}{4}$

Solution : (2) f(x) 4x + 3

Applying the concept of inverse of a function

$$\frac{y-3}{4}=x$$

$$\frac{x-3}{4}=f(x) \qquad \Rightarrow f(y)\frac{y-3}{4}$$

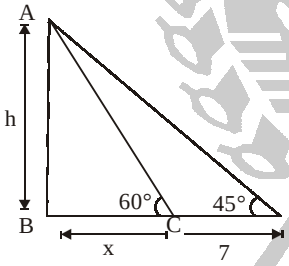
24. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that CD = 7 m. From D the angle of elevation of the point A is 45°. Then the height of the pole is

(1) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)m$ (2) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}+1}m$

(3) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}m$ (4) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)m$

Solution : (4)

Accorind to the problem In $\Delta ABC \Rightarrow \frac{h}{x}=\tan 60°$



$$\frac{h}{\sqrt{3}}=x$$

$$\Delta ABD \Rightarrow \frac{h}{x+7}=\tan 45$$

$$\Rightarrow h-\frac{h}{\sqrt{3}}=7$$

$$h\frac{\left(\sqrt{3}-1\right)}{\sqrt{3}}=7$$

$$h=\frac{7\sqrt{3}}{\sqrt{3}-1}$$

$$h=\frac{7\sqrt{3}}{2}\times(\sqrt{3}+1)$$

25. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then P(A ∪ B) is

(1) 1 (2) $\frac{2}{5}$

(3) $\frac{3}{5}$ (4) 0

Solution : (1) Event A {4, 5, 6} Event {1, 2, 3, 4}

P(A) = 3/6 = 1/2 P(B) = 4/6 = 2/3

$$(A\cap B)=\frac{1}{6}$$

$$P(A\cup B)=P(A)+(B)=\frac{1}{2}+\frac{2}{3}=\frac{7}{6}-\frac{1}{6}=1$$

26. It is given that the events A and B are such that P(A) = $\frac{1}{4}$, P(A | B) = $\frac{1}{2}$ and (P(B | A) = $\frac{2}{3}$. Then P(B) is

(1) $\frac{2}{3}$ (2) $\frac{1}{2}$

(3) $\frac{1}{6}$ (4) $\frac{1}{3}$

Solution : (4)

P(A) = 1/4 P(A/B) = 1/2 P(B/A) = 2/3

P(A/B) = $\frac{P(A\cap B)}{P(B)}$ & P(B/A) = $\frac{P(B\cap A)}{P(A)}$

$$\frac{1}{2}.P(B)=\frac{2}{3}.P(A)$$

$$\frac{1}{4}.P(B)=\frac{2}{3}\times\frac{1}{4}$$

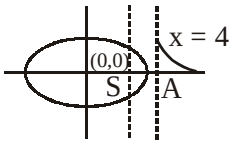
P(B) = 1/3

27. A focus of an ellipse is at the origin. The directrix is the line x = 4 and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is

(1) $\frac{4}{3}$ (2) $\frac{5}{3}$

(3) $\frac{8}{3}$ (4) $\frac{2}{3}$

Solution : (3)



According to general equation of curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ we know distance between directrix & focus is = $a\left(\frac{1}{e}-e\right)$

Hence $a\left(\frac{1}{e}-e\right)=4$

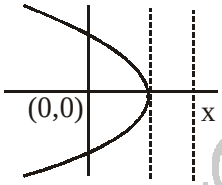
$$a\left(\frac{1}{(1/2)}-\frac{1}{2}\right)=4 \Rightarrow a(3/2)=4 \Rightarrow a=8/3$$

28. A parabola has the origin as its focus and the line x = 2 as the directrix. Then the vertex of the parabola is at

(1) (0, 1) (2) (2, 0)
(3) (0 , 2) (4) (1 , 0)

Solution : (4)

Focus (0, 0) directrix x = 2



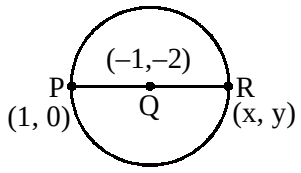
We know vertex is mid pt of focus & pt of intersection of directrix

$\therefore$ vertex = (1, 0)

29. The point diametrically opposite to the point P(1, 0) on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is

(1) (–3, –4) (2) (3, 4)
(3) (3, –4) (4) (–3, 4)

Solution : (1) Centre of circle is (–1, –2) = 0



P (1, 0) Q(x, y)

Since O is mid point of P Q

Hence $\frac{1+x}{2}=-1$ $\Rightarrow x = -3$

$\frac{O+y}{1}=-2$ $\Rightarrow y = -4$

30. The perpendicular bisector of the line segment joining P(1, 4) and Q(k, 3) has y-intercept –4. Then a possible value of k is

(1) –2 (2) –4
(3) 1 (4) 2

Solution : (2) Slope of PQ = $\frac{3-4}{k-1}=\frac{-1}{k-1}$

Slope of bisector of PQ is (k – 1)

Equation of perpendicular bisector

= y = (k – 1) x + C .... (1)

Since mid point PQ also lies on O

Mid point of P is $\left(\frac{K+1}{2}, \frac{7}{2}\right)$

$\therefore$ $\frac{7}{2}=\frac{(k+1)}{2}(k-1)+C$

$\frac{7}{2}=\frac{k^2-1}{2}+(-4)$ (given C = –4)

$7 = k^2 - 1 - 8$

$k^2 = 16$

$k = \pm 4$

k = –4 (Answer)

31. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is

(1) 12 (2) 4
(3) –4 (4) –12

Solution : (4) Let G.P. is a, ar, ar^2.....

According to condition

$a(1 + r) = 12$ .... (1)

$ar^2 + ar^3 = 48$ .... (2)

$ar^2 (1+r) = 48$

$\Rightarrow r^2 (12) = 48$

$r = \pm 2$

Since terms of G.P are alternatively positive and negative

$\therefore$ r = –2

First term is evaluated by equation (1)

$a(1 – 2) = 12$

a = –12

32. Suppose the cubic $x^3 - px + q$ has three distinct real roots where p > 0 and q > 0. Then which one of the following holds?

(1) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

(2) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

(3) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$

(4) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$

Solution : (3) Let $y = x^3 - px + q$

$$\frac{dy}{dx} = 3x^2 - p$$

For maxima or minima $\frac{dy}{dx} = 0$

$\Rightarrow x = \pm \sqrt{\frac{p}{3}}$

$$\frac{d^2y}{dx^2} = 6x$$

$$\frac{d^2y}{dx^2} > 0 \text{ at } x = \sqrt{\frac{p}{3}}$$

$$\frac{d^2y}{dx^2} < 0 \text{ at } x = -\sqrt{\frac{p}{3}}$$

$\therefore$ y has maxima at $x = x = -\sqrt{\frac{p}{3}}$

and minima at $x = \sqrt{\frac{p}{3}}$

33. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?

(1) 3 (2) 5

(3) 7 (4) 1

Solution : (4) Let $y = x^7 + 14x^5 + 16x^3 + 30x - 560$

$$\frac{dy}{dx} = 7x^6 + 70x^4 + 48x^2 + 30 > 0 \ \forall \ x \in R$$

Hence y is strictly increasing function.

Also odd degree function has range 'R'

$\therefore$ Function $x^3 + 14x^5 + 16x^3 + 30x - 560$ will intersect exactly at one point on x-axis

Hence there is only one solution.

34. Let $f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1} & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$

The which one of the following is true?

(1) f is differentiable at x = 0 but not at x = 1

(2) f is differentiable at x = 1 but not at x = 0

(3) f is neither differentiable at x = 0 nor at x = 1

(4) f is differentiable at x = 0 and at x = 1

Solution : (1) $f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1} & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$

f(x) is continuous at x = 0 at x=1

$$\therefore f'(x) = \sin\frac{1}{x-1} - \frac{1}{(x-1)} \cdot \cos\frac{1}{(x-1)}$$

$\Rightarrow f'(0^+) = -\sin 1 + \cos 1$

and $f'(0^-) = -\sin 1 + \cos 1$

$\therefore$ differentiable at x = 0

and $f'(1^+)$ = Not defined

$f'(1^-)$ = Not defined

function is not differentiable at x = 1

35. The solution the differential equation

$$\frac{dy}{dx} = \frac{x+y}{x}$$

satisfying the condition y(1) = 1 is

(1) $y = x\, e^{(x-1)}$ (2) $y = x\,ln\, x + x$

(3) $y = ln\, x + x$ (4) $y = x\, ln x + x^2$

Solution : (2) $\frac{dy}{dx} = \frac{x+y}{x}$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = 1$$

This is linear differention equation

I.f. $= e^{-\int \frac{1}{x} dx} = e^{-\log_e x} = \frac{1}{x}$

$\therefore$ solution is $y \times \frac{1}{x} = \int 1 \times \frac{1}{x} dx$

$$\Rightarrow \frac{y}{x} = \log_e x + C$$

given y(1) = 1 $\Rightarrow$ e = 1

$$\therefore \frac{y}{x} = \log_e x + 1$$

$$\Rightarrow y = x \log_e x + x$$

CHEMISTRY

36. Which one of the following is the correct statement?
    (1) Chlorides of both beryllium and aluminium have bridged chloride structures in solid phase
    (2) $B_2H_6 \cdot 2NH_3$ is known as inorganic benzene
    (3) Boric acid is a protonic acid
    (4) Beryllium exhibits coordination number of six

Solution : (1) Factual

37. The treatment of CH_3MgX with $CH_3C \equiv C–H$ produces

    (1) $CH_3-\overset{H}{\underset{|}{C}}=\overset{H}{\underset{|}{C}}-CH_3$ (2) CH_4
    (3) $CH_3–CH=CH_2$ (4) $CH_3C \equiv C-CH_3$

Solution : (2)

$$CH_3–Mg–X + CH_3–C \equiv C–H \longrightarrow CH_4 + CH_3–C \equiv C–MgX$$

38. The correct decreasing order of priority for the functional groups of organic compounds in the IUPAC system of nomenclature is
    (1) –CHO, –COOH, $–SO_3H$, $–CONH_2$
    (2) $–CONH_2$, –CHO, $–SO_3H$, –COOH
    (3) –COOH, $–SO_3H$, $–CONH_2$, –CHO
    (4) $–SO_3H$, –COOH, $–CONH_2$, –CHO

Solution : (3)

–COOH > $–SO_3H$ > $–CONH_2$ > –CHO

(NCERT based)

39. The pKa of a weak acid, HA, is 4.80. The pKb of a weak base, BOH, is 4.78. The pH of an aqueous solution of the corresponding salt, BA, will be
    (1) 7.01 (2) 9.22
    (3) 9.58 (4) 4.79

Solution : (1)

For a salt of weak acid and weak base

$$pH = 7 + \frac{1}{2}[pKa - pKb]$$

$$= 7 + \frac{1}{2}[4.80 - 4.78] = 7 + \frac{1}{2}[0.02] = 7.01$$

40. The hydrocarbon which can react with sodium in liquid ammonia is
    (1) $CH_3CH=CHCH_3$
    (2) $CH_3CH_2C \equiv CCH_2CH_3$
    (3) $CH_3CH_2CH_2C \equiv CCH_2CH_2CH_3$
    (4) $CH_3CH_2C \equiv CH$

Solution : (4)

$CH_3 - CH_2 - C \equiv C - H + Na \rightarrow CH_3 - CH_2 - C \equiv C^{\odot}Na^{\oplus}$. The terminal carbon of $–C \equiv C–$ has an acidic H. This acidic H reacts with metals which are basic in nature

41. Given $E^{\circ}_{Cr^{3+}/Cr} = –0.72$ V, $E^{\circ}_{Fe^{2+}/Fe} = –0.42$ V. The potential for the cell
    $Cr \mid Cr^{3+}$ (0.1 M) $\| Fe^{2+}$ (0.01 M) $\mid Fe$ is
    (1) –0.339 V (2) –0.26 V
    (3) 0.26 V (4) 0.339 V

Solution : (3)

$Cr \mid Cr^{+3} \| Fe^{+2} \mid Fe$

(0.1 M) (0.01M)

According to Nernst equation

$$E = \left(E^{\circ}_{Cr/Cr^{+3}} + E^{\circ}_{Fe^{+2}/Fe}\right) - \frac{0.059}{6}\log\frac{\left[Cr^{+3}\right]^2}{\left[Fe^{+2}\right]^3}$$

$$= \left[0.72 + (-0.42)\right] - \frac{0.059}{6}\log\frac{(0.1)^2}{(0.01)^3}$$

$$= 0.30 - 0.01\log\frac{10^{-2}}{10^{-6}} = 0.30 - 0.01\log 10^4$$

$= 0.30 – 0.04 = 0.26$ V

42. Amount of oxalic acid present in a solution can be determined by its titration with $KMnO_4$ solution in the presence of H_2SO_4. The titration gives unsatisfactory result when carried out in the presence of HCl, because HCl
    (1) Reduces permanganate to Mn^{2+}
    (2) Oxidises oxalic acid to carbon dioxide and water
    (3) Gets oxidised by oxalic acid to chlorine
    (4) Furnishes H^+ ions in addition to those from oxalic acid

Solution : (1) Cl^- gets oxidised in preference over oxalic acids thereby reduces $KMnO_4$ to Mn^{+2}.

43. Among the following substituted silanes the one which will give rise to cross linked silicone polymer on hydrolysis is

(1) R_2SiCl_2 (2) R_3SiCl

(3) R_4Si (4) $RSiCl_3$

Solution : (1) Factual

44. Oxidising power of chlorine in aqueous solution can be determined by the parameters indicated below :

$$\frac{1}{2}Cl_2(g) \xrightarrow{\frac{1}{2}\Delta_{diss}H^{\odot}} Cl(g) \xrightarrow{\Delta_{eg}H^{\odot}} Cl^-(g)$$

$$\xrightarrow{\Delta_{hyd}H^{\odot}} Cl^-(aq)$$

The energy involved in the conversion of

$\frac{1}{2}Cl_2(g)$ to $Cl^-(aq)$

(using the data,

$\Delta_{diss}H^{\odot}_{Cl_2} = 240$ kJ mol^{-1}

$\Delta_{eg}H^{\odot}_{Cl} = -349$ kJ mol^{-1}

$\Delta_{hyd}H^{\odot}_{Cl^-} = -381$ kJ mol^{-1} will be

(1) –850 kJ mol^{-1} (2) +120 kJ mol^{-1}

(3) +152 kJ mol^{-1} (4) –610 kJ mol^{-1}

Solution : (4)

ΔH for $\frac{1}{2}Cl_2(g) \rightarrow Cl^-(aq)$, is

$$\Delta H = \frac{1}{2}\Delta_{diss}H^{\odot} + \Delta egH^{\odot} + \Delta hydH^{\odot}$$

$$= \frac{1}{2} \times 240 - 349 - 381 = -610 \text{ kJ/mol}$$

45. Which of the following factors is of no significance for roasting sulphide ores to the oxides and not subjecting the sulphide ores to carbon reduction directly?

(1) Metal sulphides are less stable than the corresponding oxides

(2) CO_2 is more volatile than CS_2

(3) Metal sulphides are thermodynamically more stable than CS_2

(4) CO_2 is thermodynamically more stable than CS_2

Solution : (2) Factual

46. Four species are listed below :

i. HCO_3^-

ii. H_3O^+

iii. HSO_4^-

iv. HSO_3F

Which one of the following is the correct sequence of their acid strength ?

(1) i < iii < ii < iv (2) iii < i < iv < ii

(3) iv < ii < iii < i (4) ii < iii < i < iv

Solution : (1)

$HSO_3F > H_3O^+ > HSO_4^- > HCO_3^-$

iv > ii > iii > i

47. Which one of the following constitutes a group of the isoelectronic species ?

(1) CN^-, N_2, O_2^{2-}, C_2^{2-}

(2) N_2, O_2^-, NO^-, CO

(3) C_2^{2-}, O_2^-, CO, NO

(4) NO^+, C_2^{2-}, CN^-, N_2

Solution : (4)

NO^+, C_2^{2-}, CN^- & N_2 all have 14 e^- each and all show similar molecular configuration

$$\sigma 1s^2 < \sigma^* 1s^2 < \sigma 2s^2 < \sigma^* 2s^2 < \frac{\pi^2 px^2}{\pi 2py^2} < \sigma 2pz^2$$

48. Phenol, when it first reacts with concentrated sulphuric acid and then with concentrated nitric acid, gives

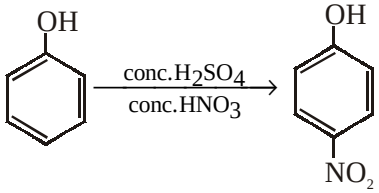
(1) p-nitrophenol

(2) Nitrobenzene

(3) 2, 4, 6-trinitrobenzene

(4) o-nitrophenol

Solution : (1)



49. The ionization enthalpy of hydrogen atom is 1.312×10^6 J mol^{-1}. The energy required to excite the electron in the atom from n = 1 to n = 2 is

(1) 7.56×10^5 Jmol^{-1}

(2) 9.84×10^5 Jmol^{-1}

(3) 8.51×10^5 Jmol^{-1}

(4) 6.56×10^5 Jmol^{-1}

Solution : (2)

$$E = -1.312 \times 10^6 \text{ J} \times \frac{Z^2}{n^2}$$

$$E_1 = -1.312 \times 10^6 \times \frac{1}{1}$$

$$E_2 = -1.312 \times 10^6 \times \frac{1}{4}$$

$= -0.328 \times 10^6$.

Energy required to excite e^- from n = 1 to n = 2,

$E_2 - E_1 = [-0.328 \times 10^6] - [1.312 \times 10^6]$

$= 9.84 \times 10^5$ J/mol

50. The organic chloro compound, which shows complete stereochemical inversion during a S_N2 reaction, is

(1) $(CH_3)_2CHCl$

(2) CH_3Cl

(3) $(C_2H_5)_2CHCl$

(4) $(CH_3)_3CCl$

Solution : (2)

Ligher the group, faster is the reactivity towards S_N2 thus complete stereochemical inversion.

51. Toluene is nitrated and the resulting product is reduced with tin and hydrochloric acid. The product so obtained is diazotised and then heated with cuprous bromide. The reaction mixture so formed contains

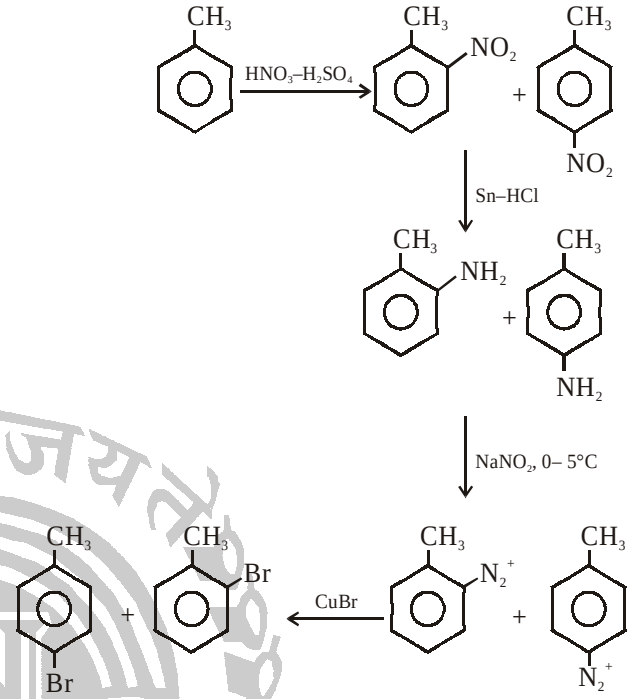
(1) Mixture of o- and p-bromoanilines

(2) Mixture of o- and m-bromotoluenes

(3) Mixture of o- and p-bromotoluenes

(4) Mixture of o- and p=dibromobenzenes

Solution : (3)



52. In the following sequence of reactions, the alkene affords the compound B'

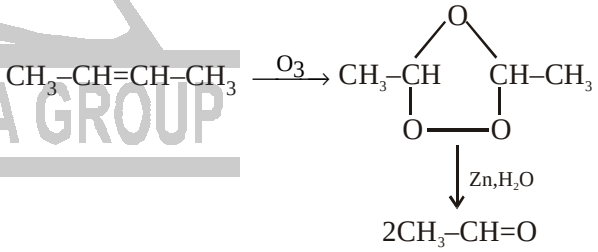
$$CH_3CH=CHCH_3 \xrightarrow{O_3} A \xrightarrow[Zn]{H_2O} B$$

The compound B is

(1) $CH_3CH_2COCH_3$ (2) CH_3CHO

(3) CH_3CH_2CHO (4) CH_3COCH_3

Solution : (2)



53. Which one of the following pairs of species have the same bond order?

(1) O_2^- and CN^- (2) NO^+ and CN^+

(3) CN^- and NO^+ (4) CN^- and CN^+

Solution : (3)

CN^- & NO^+ both have $14e^-$.

Bond order $= \frac{1}{2}[10-4] = 3$

54. At 80°C , the vapour pressure of pure liquid 'A' is 520 mm Hg and that of pure liquid 'B' is 1000 mm Hg. If a mixture solution of 'A' and 'B' boils at 80°C and 1 atm pressure, the amount of 'A' in the mixture is (1 atm = 760 mm Hg)

(1) 48 mol percent (2) 50 mol percent

(3) 52 mol percent (4) 34 mol percent

Solution : (2) According to Raoult's law,

$P_T = p_A + p_B$

$760 = 520\, x_A + 1000\,(1-x_A) = -480x_A + 1000$

$480x_A = 240$

$x_A = 0.5$

Thus, A and B both are 50 mol percent

55. For a reaction $\frac{1}{2}A \rightarrow 2B$, rate of disappearance of 'A' is related to the rate of appearance of 'B' by the expression

(1) $-\frac{d[A]}{dt} = \frac{d[B]}{dt}$ (2) $-\frac{d[A]}{dt} = 4\frac{d[B]}{dt}$

(3) $-\frac{d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt}$ (4) $-\frac{d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$

Solution : (4)

$\frac{1}{2}A \rightarrow 2B$

$-2\frac{d[A]}{dt} = +\frac{1}{2}\frac{d[B]}{dt}$

$-\frac{d[A]}{dt} = +\frac{1}{4}\frac{d[B]}{dt}$

56. The equilibrium constants K_{P_1} and K_{P_2} for the reactions $X \rightleftharpoons 2Y$ and $Z \rightleftharpoons P + Q$, respectively are in the ratio of 1 : 9. If the degree of dissociation of X and Z be equal then the ratio of total pressures at these equilibria is

(1) 1 : 3 (2) 1 : 9

(3) 1 : 36 (4) 1 : 1

Solution : (3)

$X \rightleftharpoons 2Y$			$Z = P + Q$		
Int	1	–	Int	1	–	–
At eq.	$1-\alpha$	2α	At. eq.	$1-\alpha$	α	α

$K_{P_1} = \frac{4\alpha^2}{1-\alpha} \times \left[\frac{P_1}{1+\alpha}\right]$ $K_{P_2} = \frac{\alpha^2}{1-\alpha} \times \left[\frac{P_2}{1+\alpha}\right]$

Non, $\frac{K_{P_1}}{K_{P_2}} = \frac{1}{9} = \frac{4\alpha^2 \times P_1}{\alpha^2 \bullet P_2}$

$\therefore \frac{P_1}{P_2} = \frac{1}{36}$

57. In context with the industrial preparation of hydrogen from water gas ($CO+H_2$), which of the following is the correct statement ?

(1) H_2 is removed through occlusion with Pd

(2) CO is oxidised to CO_2 with steam in the presence of a catalyst followed by absorption of CO_2 in alkali

(3) CO and H_2 are fractionally separated using differences in their densities

(4) CO is removed by absorption in aqueous Cu_2Cl_2 solution

Solution : (2)

Factual

58. In which of the following octahedral complexes of Co (at. no. 27), will the magnitude of Δ_0 be the highest?

(1) $[Co(H_2O)_6]^{3+}$

(2) $[Co(NH_3)_6]^{3+}$

(3) $[Co(CN)_6]^{3-}$

(4) $[Co(C_2O_4)_3]^{3-}$

Solution : (3)

CN^- is the strongest field ligand & thus Δ_0 is maximum

59. The coordination number and the oxidation state of the element 'E' in the complex $[E(en)_2(C_2O_4)]NO_2$ (where (en) is ethylene diamine) are, respectively

(1) 4 and 3 (2) 6 and 3

(3) 6 and 2 (4) 4 and 2

Solution : (2)

$[E(en)_2(C_2O_4)]NO_2$

Co.No. $\longrightarrow 6$

Oxd. state $\longrightarrow x + 0 + (-2) + (-1) = 0$

$\Rightarrow x = +3$

60. Identify the wrong statement in the following :
(1) Ozone layer does not permit infrared radiation from the sun to reach the earth
(2) Acid rain is mostly because of oxides of nitrogen and sulphur
(3) Chlorofluorocarbons are responsible for ozone layer depletion
(4) Greenhouse effect is responsible for global warming

Solution : (1)

Ozone layer does not permit U.V. radiation from entering the earth's atmosphere

61. Larger number of oxidation states are exhibit by the actinoids than those by the lanthanoids the main reason being
(1) More energy difference between 5f and than between 4f and 5d orbitals
(2) More reactive nature of the actinoids than the lanthanoids
(3) 4f orbitals more diffused than the 5f orbitals
(4) Lesser energy difference between 5f and 6d than between 4f and 5d orbitals

Solution : (3)

Since, 4f orbitals are more diffused in the atom, not much of the electrons can be excited

62. In a compound, atoms of element Y form ccp lattice and those of element X occupy 2/3rd of tetrahedral voids. The formula of the compound will be
(1) X_2Y (2) X_3Y_4
(3) X_4Y_3 (4) X_2Y_3

Solution : (3)

$$Y \rightarrow 8\times\frac{1}{8}+6\times\frac{1}{2}=4$$

$$X \rightarrow \frac{2}{3}+8=\frac{16}{3}$$

$$X_{\frac{16}{3}}Y_4 \Rightarrow X_{\frac{4}{3}}Y_1 \Rightarrow X_4Y_3$$

63. Gold numbers of protective colloids A, B, C and D are 0.50, 0.01, 0.10 and 0.005, respectively. The correct order of their protective powers is
(1) A < C < B < D (2) B < D < A< C
(3) D < A < C < B (4) C < B < D < A

Solution : (1)

$$\text{Protective power} \propto \frac{1}{\text{Gold No.}}$$

Protective power A < C < B < D

Gold No. 0.5 < 0.10 < 0.01 < 0.005

64. The vapour pressure of water at 20°C is 17.5 mm Hg. If 18g of glucose ($C_6H_{12}O_6$) is added to 178.2g of water at 20°C, the vapour pressure of the resulting solution will be
(1) 16.500 mm Hg (2) 17.325 mm Hg
(3) 17.675 mm Hg (4) 15.750 mm Hg

Solution : (2)

$$n_{glucose}=\frac{18}{180}=0.1$$

$$n_{H_2O}=\frac{178.2}{18}=9.9$$

$$x_{H_2O}=\frac{9.9}{10}=0.99$$

$$x_{glucose}=\frac{0.1}{10}=0.01$$

$$P_T=P^0_{H_2O}\times x_{H_2O}+P^0_{Glucose}\times x_{Glucose}$$

$= 17.5 \times 0.99 + 0$

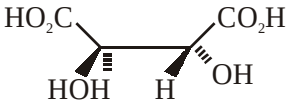
$= 17.325$ mm Hg

65. Bakelite is obtained from phenol by reacting with
(1) CH_3COCH_3 (2) HCHO
(3) $(CH_2OH)_2$ (4) CH_3CHO

Solution : (2)

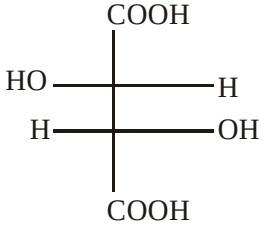
Factual

66. The absolute configuration of



(1) R, S (2) S, R
(3) S, S (4) R, R

Solution : (4)



67. For the following three reactions a, b and c, equilibrium constants are given :

a. $CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g); K_1$

b. $CH_4(g) + H_2O(g) \rightleftharpoons CO(g) + 3H_2(g); K_2$

c. $CH_4(g) + 2H_2O(g) \rightleftharpoons CO_2(g) + 4H_2(g); K_3$

Which of the following relations is correct ?

(1) $K_3 = K_1K_2$

(2) $K_3 \bullet K_2^3 = K_1^2$

(3) $K_1\sqrt{K_2} = K_3$

(4) $K_2K_3 = K_1$

Solution : (1)

$K_3 = K_1 \times K_2$

$$\frac{[CO_2][H_2]^4}{[CH_4][H_2O]^2} = \frac{[H_2][CO_2]}{[CO][H_2O]} \times \frac{[CO][H_2]^3}{[CH_4][H_2O]}$$

L.H.S. = R.H.S.

68. Standard entropy of X_2, Y_2 and XY_3 are 60, 40 and 50 JK^{-1} mol^{-1}, respectively. For the reaction, $\frac{1}{2}X_2 + \frac{3}{2}Y_2 \rightarrow XY_3 \quad \Delta H = -30kJ$, to be at equilibrium, the temperature will be

(1) 750 K

(2) 1000 K

(3) 1250 K

(4) 500 K

Solution : (1)

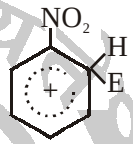
$$\Delta S_{R^n} = S_P^\circ - S_R^\circ$$

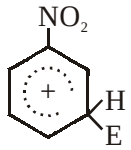
$$= [50] - \left[\frac{1}{2} \times 60 + \frac{3}{2} \times 40\right] = 50 - [30 + 60] = -40 \text{ J}$$

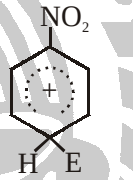
$$\Delta S = \frac{\Delta H}{T}$$

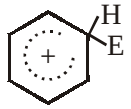
$$T = \frac{-30,000}{-40} = 750K$$

69. The electrophile, $E^\oplus$ attacks the benzene ring to generate the intermediate σ-complex. Of the following, which σ -complex is of lowest energy?

(1) 

(2) 

(3) 

(4) 

Solution : (2)

NO_2 – is a meta directing group & thus, the electrophile attaches on meta group, with max, stability & lowest energy

70. α-D-(+)-glucose and β-D-(+)-glucose are

(1) Anomers (2) Enantiomers

(3) Conformers (4) Epimers

Solution : (1)

α – D(+) glucose & B–D–(+) glucose are called anomers

PHYSICS

71. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1 : Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion.

and

Statement-2 : For heave nuclei, binding energy per nucleon increases with increasing Z while for light nuclei it decreases with increasing Z.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is false, Statement-2 is true

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Key : (2) Theoretical (B.E. per nucleaon curve)

72. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1 : For a mass M kept at the centre of a cube of side 'a', the flux of gravitational field passing through its sides is 4π GM.

and

Statement-2 : If the direction of a field due to a point source is radial and its dependence on the distance 'r' from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is false, Statement-2 is true

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

Key : (4) Gravitational flux through cube = 4π G (Mass enclosed for inverse square law force)

73. Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, its is found that the screw gauge has a zero error of –0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is

(1) 3.67 mm (2) 3.38 mm

(3) 3.32 mm (4) 4.73 mm

Key : (2) Diameter = $3+\frac{35}{100}+0.03\,\text{mm}$

74. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure P_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be

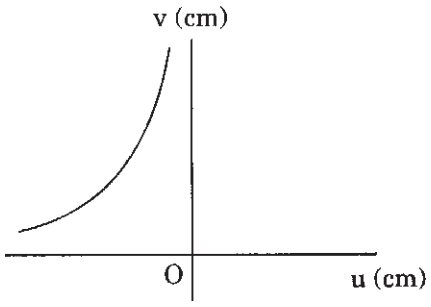
(1) $\frac{P_1V_1T_2+P_2V_2T_1}{P_1V_1+P_2V_2}$ (2) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_1+P_2V_2T_2}$

(3) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_2+P_2V_2T_1}$ (4) $\frac{P_1V_1T_1+P_2V_2T_2}{P_1V_1+P_2V_2}$

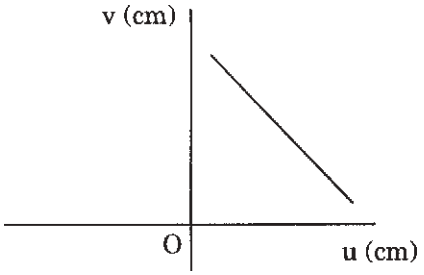
Key : (3) $\frac{P_1V_1}{T_1}+\frac{P_2V_2}{T_2}=\frac{(P_1V_1+P_2V_2)}{T}$

75. A student measures the focal length of a convex lens by putting an object pin at a distance 'u' from the lens and measuring the distance 'v' of the image pin. The graph between 'u' and 'v' plotted by the student should look like

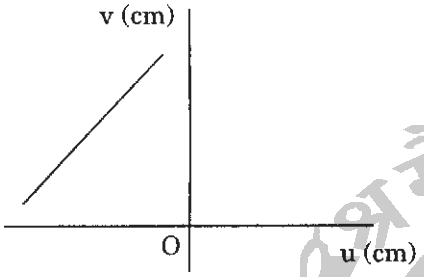
(1)



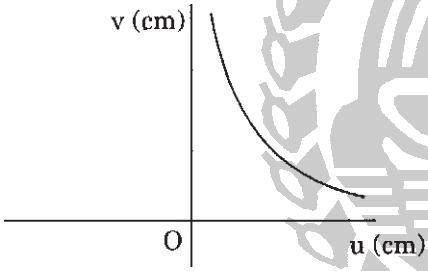
(2)



(3)



(4)



Key : (1) $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

Directions : *Questions No. 76 and 77 are based on the following paragraph.*

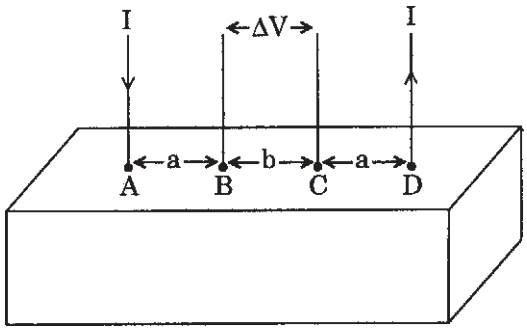
Consider a block of conducting material of resistivity ' ρ ' shown in the figure. Current 'I' enters at 'A' and leaves from 'D'. We apply superposition principle to find voltage ' ΔV ' developed between 'B' and 'C'. The calculation is done in the following steps:

(i) Take current 'I' entering from 'A' and assume it to spread over a hemispherical surface in the block.

(ii) calculate field E(r) at distance 'r' from A by using Ohm's law $E = \rho j$, where 'j' is the current per unit area at 'r'.

(iii) From the 'r' dependence of E(r), obtain the potential V(r) at r.

(iv) Repeat (i), (ii) and (iii) for current 'I' leaving 'D' and superpose results for 'A' and 'D'.



76. ΔV measured between B and C is

(1) $\frac{\rho I}{2\pi a}-\frac{\rho I}{2\pi(a+b)}$ (2) $\frac{\rho I}{2\pi(a-b)}$

(3) $\frac{\rho I}{\pi a}-\frac{\rho I}{\pi(a+b)}$ (4) $\frac{\rho I}{a}-\frac{\rho I}{(a+b)}$

Key : (1) $\Delta V=\int E\cdot dr=\int_{a}^{a+b}\frac{\rho I}{2\pi r^2}dr$

77. For current entering at A, the electric field at a distance 'r' from A is

(1) $\frac{\rho I}{2\pi r^2}$ (2) $\frac{\rho I}{4\pi r^2}$

(3) $\frac{\rho I}{8\pi r^2}$ (4) $\frac{\rho I}{r^2}$

Key : (1) $E = \rho J = \rho\cdot\frac{I}{2\pi r^2}$

78. Consider a uniform square plate of side 'a' and mass 'm'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

(1) $\frac{7}{12}ma^2$ (2) $\frac{2}{3}ma^2$

(3) $\frac{5}{6}ma^2$ (4) $\frac{1}{12}ma^2$

Key : (2) $I = I_0 + Md^2$

$$=\left(\frac{Ma^2}{12}+\frac{Ma^2}{12}\right)+M\cdot\left(\frac{a}{\sqrt{2}}\right)^2=\frac{2}{3}Ma^2$$

79. An experiment is performed to find the refractive index of glass using a travelling microscope. In this experiment, distances are measured by
(1) a meter scale provided on the microscope
(2) a screw gauge provided on the microscope
(3) a vernier scale provided on the microscope
(4) a standard laboratory scale

Key : (3) Informative

80. A horizontal overhead power-line is at a height of 4 m from the ground and carries a current of 100 A from east to west. The magnetic field directly below it on the ground is ($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})
(1) 5×10^{-6} T southward
(2) 2.5×10^{-7} T northward
(3) 2.5×10^{-7} T southward
(4) 5×10^{-6} T northward

Key : (1) $B = \frac{\mu_0}{4\pi}.\frac{I}{d}(\sin 90° + \sin 90°)$, Southward

$= \frac{\mu_0 I}{2\pi d} = 5 \times 10^{-6}$T, Southward

81. The speed of sound in oxygen (O_2) at a certain temperature is 460 ms^{-1}. The speed of sound in helium (He) at the same temperature will be (assume both gases to be ideal)
(1) 650 ms^{-1} (2) 330 ms^{-1}
(3) 460 ms^{-1} (4) 500 ms^{-1}

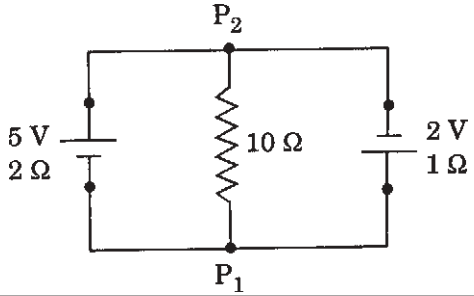
Key : (No Answer)

$$V = \sqrt{\frac{\gamma RT}{M}}$$

$$\frac{V_{He}}{V_{O_2}} = \sqrt{\frac{\gamma_{He}}{\gamma_{O_2}} \times \frac{M_{O_2}}{M_{He}}}$$

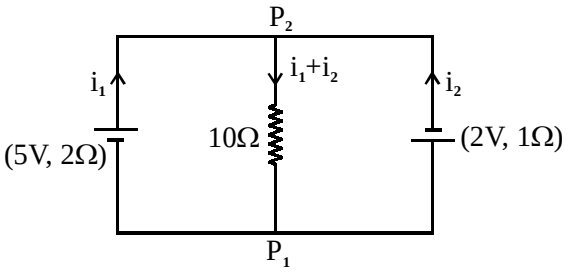
No correct option

82. A 5 V battery with internal resistance 2 Ω and a 2V battery with internal resistance 1 Ω are connected to a 10 Ω resistor as shown in the figure.



The current in the 10 Ω resistor is
(1) 0.03 A P_2 to P_1 (2) 0.27 A P_1 to P_2
(3) 0.27 A P_2 to P_1 (4) 0.03 A P_1 to P_2

Key : (1)



$12i_1 + 10i_2 = 5$ ... (i)
$10i_1 + 11i_2 = -2$ ... (ii)
Solving $(i_1 + i_2) = 0.03$ A

83. A body of mass m = 3.513 kg is moving along the x-axis with a speed of 5.00 ms^{-1}. The magnitude of its momentum is recorded as
(1) 17.56 kg ms^{-1}
(2) 17.57 kg ms^{-1}
(3) 17.6 kg ms^{-1}
(4) 17.565 kg ms^{-1}

Key : (3) Theoretical

84. A working transistor with its three legs marked P, Q and R is tested using a multimeter. No conduction is found between P and Q. By connecting the common (negative) terminal of the multimeter to R and the other (positive) terminal to P or Q, some resistance is seen on the multimeter. Which of the following is true for the transistor ?
(1) It is a pnp transistor with R as emitter
(2) It is an npn transistor with R as collector
(3) It is an npn transistor with R as base
(4) It is a pnp transistor with R as collector

Key : (3) Conceptual

85. A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is
(1) 0.67 J (2) 0.34 J
(3) 0.16 J (4) 1.00 J

Key : (1)Energy loss $= \frac{1}{2} \times \frac{m_1 m_2}{m_1 + m_2}(u_1 - u_2)^2\left(1 - e^2\right)$

86. A wave travelling along the x-axis is described by the equation y(x, t) = 0.005 cos $(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are

(1) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$ (2) $\alpha = 12.50\,\pi, \beta = \frac{1\pi}{20}$

(3) $\alpha = 25.00\,\pi, \beta = \pi$ (4) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$

Key : (3) y = A sin [kx–wt]

Comparing $K = \alpha = \frac{2\pi}{\lambda}$ & $w = \beta = \frac{2\pi}{T}$

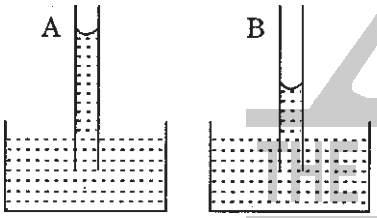
87. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area A = 10 cm^2 and length = 20 cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is ($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})

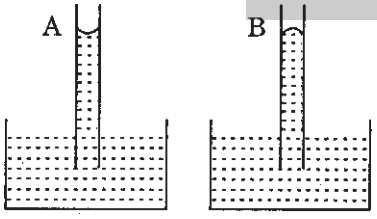
(1) $4.8\pi \times 10^{-5}$ H (2) $2.4\pi \times 10^{-4}$ H

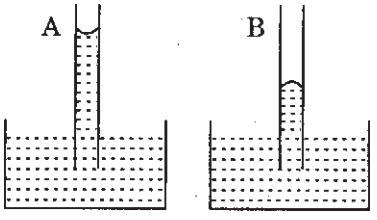
(3) $2.4\pi \times 10^{-5}$ H (4) $4.8\pi \times 10^{-4}$ H

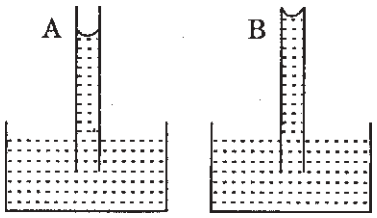
Key : (2) $M = \frac{\mu_0 N_1 N_2 A}{l}$

88. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes ?

(1) 

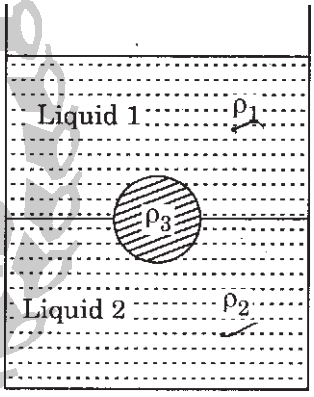
(2) 

(3) 

(4) 

Key : (1) Surface tension of water is greater than soap-water solution

89. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2, respectively. A solid ball, made of a material of density ρ_3 is dropped in the jar. It comes to equilibrium in the position shown in the figure.



Which of the following is true for ρ_1, ρ_2 and ρ_3 ?

(1) $\rho_1 < \rho_2 < \rho_3$ (2) $\rho_1 < \rho_3 < \rho_2$

(3) $\rho_3 < \rho_1 < \rho_2$ (4) $\rho_1 > \rho_3 > \rho_2$

Key : (2) $\rho_1 \frac{V}{2} g + \rho_2 \frac{V}{2} g = \rho_3 V g$

$\rho_3 = \frac{\rho_1 + \rho_2}{2}$

90. Suppose an electron is attracted towards the origin by a force $\frac{k}{r}$ where 'k' is a constant and 'r' is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the n^{th} orbital of the electron is found to be 'r_n' and the kinetic energy of the electron to be 'T_n'. Then which of the following is true ?

(1) $T_n \propto \frac{1}{n}, r_n \propto n$

(2) $T_n \propto \frac{1}{n}, r_n \propto n^2$

(3) $T_n \propto \frac{1}{n^2}, r_n \propto n^2$

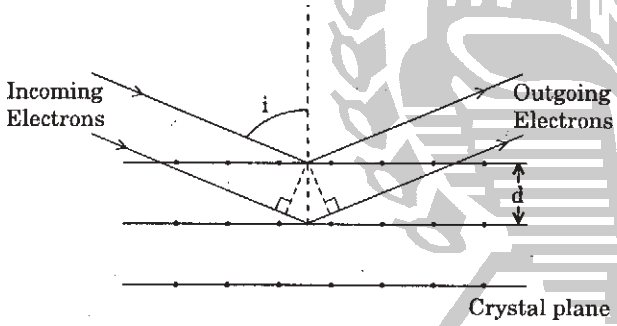
(4) T_n independent of n, $r_n \propto n$

Key : (4) $\frac{mV^2}{r} = \frac{k}{r}$

$mv^2 = k$

Directions : *Questions No. 91, 92 and 93 are based on the following paragraph.*

Wave property of electrons implies that they will show diffraction effects. Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively (see figure).



91. Electrons accelerated by potential V are diffracted from a crystal. If d = 1 Å and i = 30°, V should be about (h = 6.6×10^{-34} Js, $m_e = 9.1 \times 10^{-31}$ kg, e = 1.6×10^{-19} C)

(1) 500 V (2) 1000 V

(3) 2000 V (4) 50 V

Key : (4) $2d \sin\theta = n\lambda$

$\lambda = \frac{h}{\sqrt{2meV}}$

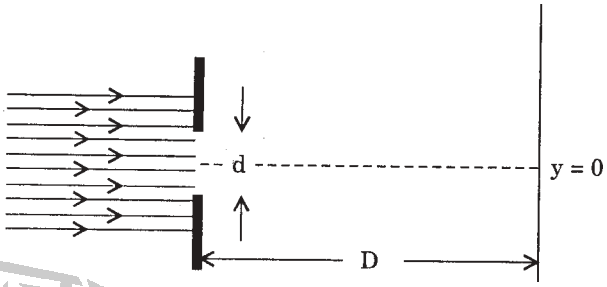
92. If a strong diffraction peak is observed when electrons are incident at an angle 'i' from the normal to the crystal planes with distance 'd' between them (see figure), de Broglie wavelength λ_{dB} of electrons can be calculated by the relationship (n is an integer)

(1) $2d \sin i = n\lambda_{dB}$ (2) $d \cos i = n\lambda_{dB}$

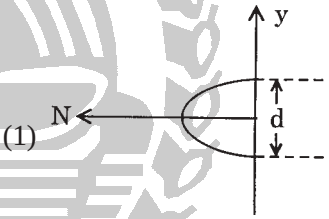
(3) $d \sin i = n\lambda_{dB}$ (4) $2d \cos i = n\lambda_{dB}$

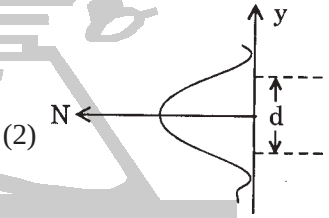
Key : (4) Theoretical

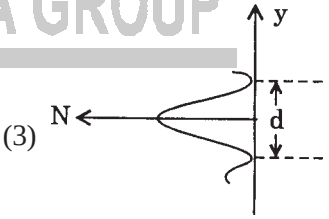
93. In an experiment, electrons are made to pass through a narrow slit of width 'd' comparable to their de Broglie wavelength. They are detected on a screen at a distance 'D' from the slit (see figure)

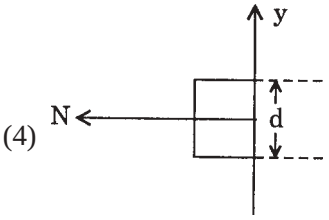


Which of the following graphs can be expected to represent the number of electrons 'N' detected as a function of the detector position 'y' (y = 0 corresponds to the middle of the slit) ?

(1)


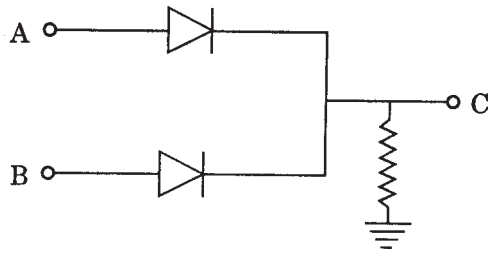
(2)


(3)


(4)


Key : (2) Theoretical

94. In the circuit below, A and B represent two inputs and C represents the output.



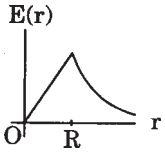
The circuit represents

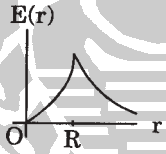
(1) NAND gate (2) OR gate

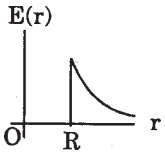
(3) NOR gate (4) AND gate

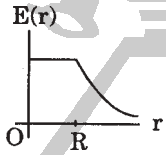
Key : (2) Theoretical

95. A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field E(r) produced by the shell in the range $0 \le r < \infty$, where r is the distance from the centre of the shell ?

(1) 

(2) 

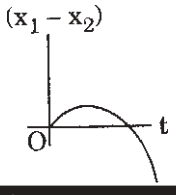
(3) 

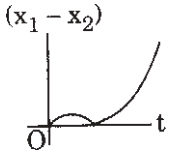
(4) 

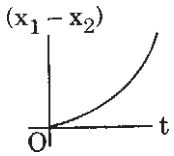
Key : (3) Electric field inside the shell is 0 and outside

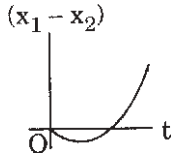
$$E \propto \frac{1}{r^2}$$

96. A body is at rest at x = 0. At t = 0, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through x = 0 moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time 't' and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time 't'?

(1) 

(2) 

(3) 

(4) 

Key : (4) At any time

$$x_1 = \frac{1}{2} at^2$$

$x_2 = vt$

$\therefore$ At any time

$$x_1 - x_2 = \frac{1}{2} at^2 - vt$$

After some time 1st body catches 2nd and that time is

$$\frac{1}{2} at^2 - vt = 0$$

or $t = \frac{2v}{a}$

97. Relative permittivity and permeability of a material are ε_r and μ_r, respectively. Which of the following values of these quantities are allowed for a diamagnetic material ?

(1) $\varepsilon_r = 0.5, \mu_r = 0.5$ (2) $\varepsilon_r = 1.5, \mu_r = 1.5$

(3) $\varepsilon_r = 0.5, \mu_r = 1.5$ (4) $\varepsilon_r = 1.5, \mu_r = 0.5$

Key : (4) $\mu_r = 1 + x$

For diag. x = –ve

$\therefore \mu_r < 1$

$$\varepsilon_r = \frac{\epsilon}{\epsilon_0}$$

For any material $\epsilon > \epsilon_0$

98. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be

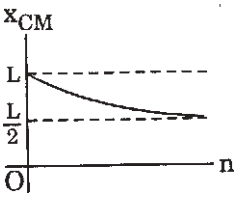
(1) 110 km s–1 (2) 0.11 km s–1

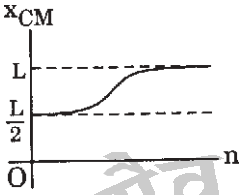
(3) 1.1 km s–1 (4) 11 km s–1

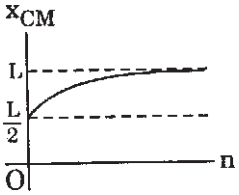
Key : (1) $V_e = \sqrt{\frac{2GM}{R}}$

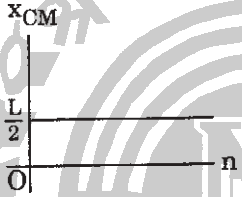
$$V_P = \sqrt{\frac{2G(10M)}{R/10}} = 10\, V_e$$

99. A thin rod of length 'L' is lying along the x-axis with its ends at x = 0 and x = L. Its linear density (mass/length) varies with x as $k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against 'n', which of the following graphs best approximates the dependence of x_{CM} on n ?

(1) 

(2) 

(3) 

(4) 

Key : (3) $X_{CM} = \frac{\int x dm}{\int dm}$

Numerator $\int x dm = \int_0^L x(\lambda dx)$

$= \frac{k}{L^n}\int_0^L x^{n+1}\,dx = \frac{KL^2}{(n+2)}$

Denominator $\int dm = \int \lambda dx$

$= \frac{K}{L^n}\int_0^L x^{n+1}dx = \frac{k}{L^n}\left(\frac{L^{n+1}}{n+1}\right) = \frac{KL}{n+1}$

$\therefore \quad X_{CM} = \left(\frac{n+1}{n+2}\right)L$

100. The dimension of magnetic field in M, L, T and C (Coulomb) is given as

(1) $M\,T^{-1}\,C^{-1}$ (2) $M\,T^{-2}\,C^{-1}$

(3) $M\,L\,T^{-1}\,C^{-1}$ (4) $M\,T^{2}\,C^{-2}$

Key : (1) Theoretical

101. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is 'd'. The space between the plates is now filled the two dielectrics. One of the dielectrics has dielectric constant $\kappa_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $\kappa_2 = 6$ and thickness $\frac{2d}{3}$. Capacitance of the capacitor is now

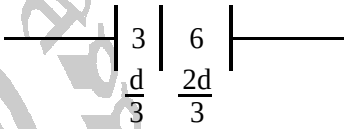
(1) 40.5 pF

(2) 20.25 pF

(3) 1.8 pF

(4) 45 pF

Key : (1)



$C = \frac{A\epsilon_0}{d} = 9$

$C_1 = \frac{3A\epsilon_0}{d/3} = 9 \times 9 = 81$

$C_2 = \frac{6A\epsilon_0}{2d/3} = 9 \times 9 = 81$

Now capacitors are in series

$\therefore \quad \frac{C_1C_2}{C_1+C_2} = 40.5$

102. An athlete in the Olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range

(1) 20,000 J – 50,000 J

(2) 2,000 J – 5,000 J

(3) 200 J – 500 J

(4) 2×10^5 J – 3×10^5 J

Key : (2) $V = \frac{100}{10} = 10$ m/s

Average mass of the man = 50 kg

$\therefore \quad K.E. = \frac{1}{2}\times 50 \times 100$

$= 2500$ J

103. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_2 ($\rho_2 < \rho_1$). Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed *v*, i.e., $F_{viscous} = kv^2$ (k > 0). The terminal speed of the ball is

(1) $\sqrt{\frac{V g \rho_1}{k}}$ (2) $\frac{V g(\rho_1 - \rho_2)}{k}$

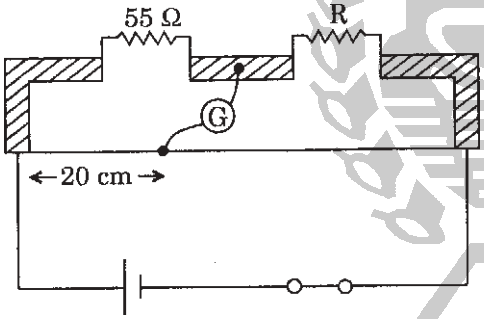
(3) $\sqrt{\frac{V g(\rho_1 - \rho_2)}{k}}$ (4) $\frac{V g \rho_1}{k}$

Key : (3) $Mg = Fu + F_{viscous}$ (At terminal velocity)

$$V\rho_1 g = V\rho_2 g + KV_0^2$$

$$\therefore \quad V_0 = \sqrt{\frac{Vg(\rho_1 - \rho_2)}{K}}$$

104. Shown in the figure below is a meter-bridge set up with null deflection in the galvanometer.



The value of the unknown resistor R is

(1) 110 Ω (2) 55 Ω

(3) 13.75 Ω (4) 220 Ω

Key : (4) $\frac{55}{20} = \frac{R}{80}$

105. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x cm for the second resonance. Then

(1) 54 > x > 36 (2) 36 > x > 18

(3) 18 > x (4) x > 54

Key : (4) $\nu = \frac{v}{4(18)}$

$$\nu = \frac{3v'}{4(x)}$$

$$\text{or} \quad \frac{v}{18} = \frac{3v'}{x}$$

$$\therefore \quad x = 54\frac{v'}{v}$$

$\therefore$ x > 54 (As v' > v)